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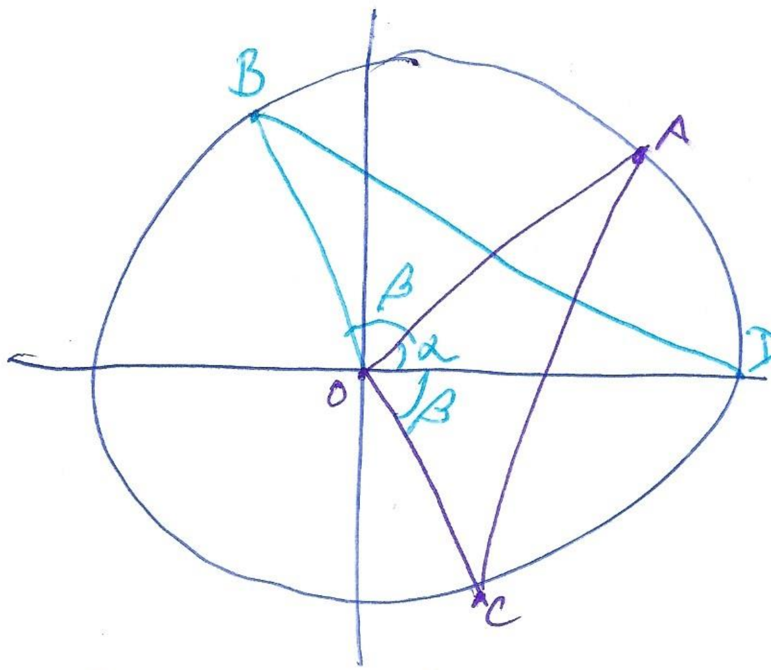

Note: $A = (\cos(\alpha), \sin(\alpha))$

$B = (\cos(\alpha+\beta), \sin(\alpha+\beta))$

$C = (\cos(-\beta), \sin(-\beta))$

$D = (1,0)$

$O = (0,0)$

1. $m\measuredangle BOD = \alpha + \beta = m\measuredangle AOC$,

   so $\overline{AC} = \overline{BD}$

2. $\sqrt{(\cos(\alpha+\beta)-1)^2+(\sin(\alpha+\beta)-0)^2} = d(B,D)$

   $= \sqrt{(\cos(\alpha)-\cos(\beta))^2+(\sin(\alpha)-\sin(-\beta))^2} = d(A,C)$

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3. $[\cos(\alpha+\beta)]^2 - 2\cos(\alpha+\beta) + 1 + [\sin(\alpha+\beta)]^2$

$= [\cos(\alpha)]^2 - 2\cos(\alpha)\cos(\beta) + [\cos(\beta)]^2 + [\sin(\alpha)]^2$

$+ 2\sin(\alpha)\sin(\beta) + [\sin(\beta)]^2.$

4. $2 - 2\cos(\alpha+\beta) = 2 + 2\sin(\alpha)\sin(\beta) - 2\cos(\alpha)\cos(\beta)$

5. $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

Ex: $\cos(5\pi/12)$

$= \cos(\pi/4 + \pi/6) = \cos(3\pi/12 + 2\pi/12)$

$\Rightarrow \cos(5\pi/12) = \cos(\pi/4)\cos(\pi/6) - \sin(\pi/4)\sin(\pi/6)$

$= (\sqrt{2}/2)(\sqrt{3}/2) - (\sqrt{2}/2)(1/2)$

$= \dfrac{\sqrt{2}(\sqrt{3}-1)}{4}$

Note: Sine and cosine are "co-functions".
Similarily, (secant and cosecant) and (tangent and cotangent) are "co-function pairs".

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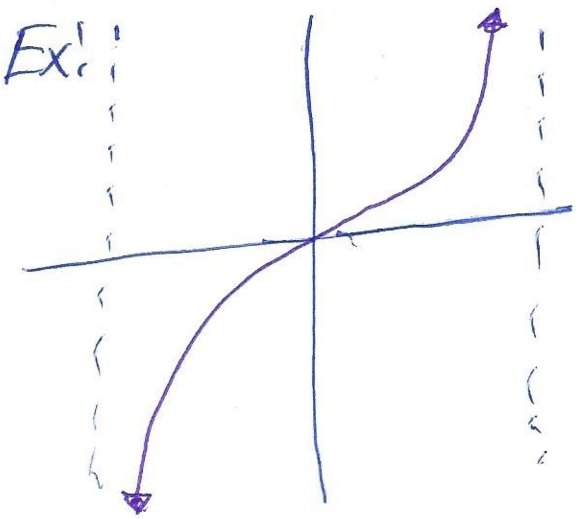
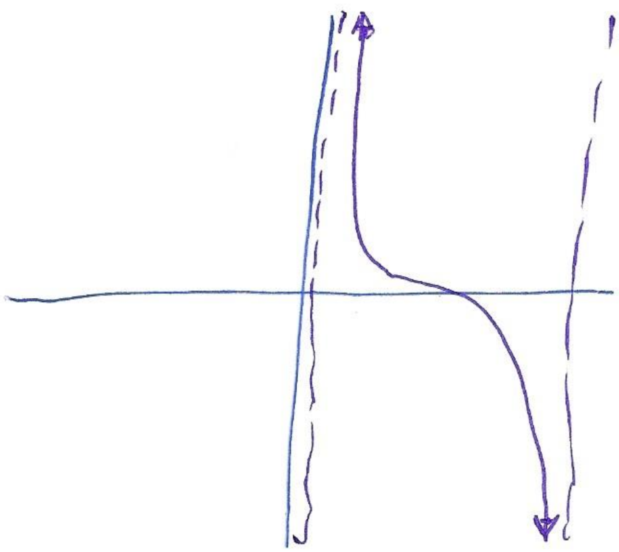
Cofunction Identity

$f(\pi/2 - \theta) = co(f)(\theta)$

Ex:



$y = \tan(x)$



$y = \cot(x)$

$$\sin(\alpha+\beta) = \cos(\pi/2 - (\alpha+\beta))$$
$$= \cos[(\pi/2 - \alpha) - \beta]$$
$$= \cos(\pi/2 - \alpha)\cos(\beta) + \sin(\pi/2 - \alpha)\sin(\beta)$$
$$= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

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$$\sin(\alpha-\beta) = \sin(\alpha)\cos(-\beta) + \cos(\alpha)\sin(-\beta)$$
$$= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$$

$$\sin(2x) = \sin(x+x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = [\cos(x)]^2 - [\sin(x)]^2$$
$$= 2[\cos(x)]^2 - 1$$
$$= 1 - 2[\sin(x)]^2$$

$$\tan(2x) = \frac{2\tan(x)}{1-[\tan(x)]^2}, \quad [\sin(x)]^2 = \frac{1-\cos(2x)}{2}$$