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#1a) \( \log_5(x) = y \)
\[ 5^y = x \]

b) \( \ln(y) = 3 \)
\[ e^3 = y \]

c) \( \ln(5) = 2 \)
\[ e^2 = 5 \]

2a) \( e^3 = y \rightarrow \ln(y) = 3 \)

b) \( u = a^v \rightarrow \log_a(u) = v \)

c) \( 10^{x+2} = r \rightarrow \log_{10}(r) = x + 2 \)

4a) \( \log(5) + \log(3) = \log(5 \cdot 3) = \log(15) \)

3) \( \ln(10) = \frac{\log_{10}}{\log_{e}} \)

Lesson 11:

\( \log_a(x) = y \)
\[ a^y = x \]

\( \log_b(a^y) = \log_b(x) \)
\( \Rightarrow y \log_b(a) = \log_b(x) \)

\( \Rightarrow y = \frac{\log_b(x)}{\log_b(a)} \)
Change of Base Formula:
\[
\log_a(x) = \frac{\log_b(x)}{\log_b(a)}
\]

4b) \( \log_3(x+2) - 2\log_3(x-1) \)
\[
= \log_3(x+2) - \log_3((x-1)^2)
\]
\[
= \log_3\left(\frac{x+2}{(x-1)^2}\right)
\]

5a) \( \log_2(xy^2z) = \log_2(x) + \log_2(y^2) + \log_2(z) \)

b) \( \log_a\left(\frac{4}{25}\right) = \log_a(4) - \log_a(25) \)
\[
= \log_a(2^2) - \log_a(5^2)
\]
\[
= 2\log_a(2) - 2\log_a(5)
\]
\[
= 2\left(\log_a(2) - \log_a(5)\right)
\]

5c) \( \ln(\sqrt{20}) = \ln\left[(20)^{\frac{1}{2}}\right] \)
\[
= \frac{1}{2}\ln(20)\]
6a) $\log_2 (x+2) + \log_2 (x-2) = 5$

$\Rightarrow \log_2 [(x+2)(x-2)] = 5$

$\Rightarrow \log_2 (x^2-4) = 5$

$\Rightarrow 2^5 = x^2 - 4$

$\Rightarrow 32 = x^2 - 4, \quad x^2 = 36$

$\Rightarrow x = \pm 6$

6b) $\log_4 (x) - \log_4 (x+2) = 2$

$\Rightarrow \log_4 \left( \frac{x}{x+2} \right) = 2$

$\Rightarrow \frac{x}{x+2} = 4^2 = 16$

$\Rightarrow x = 16x + 32$

$\Rightarrow 15x = -32$

$\Rightarrow x = \frac{-32}{15}, \text{ no solution}
6c) \( x \log(5) + x \log(7) = \log(9) \)
\[= x \left[ \log(5) + \log(7) \right] = \log(9) \]
\[= x = \frac{\log(9)}{\log(5) + \log(7)} = \frac{\log(9)}{\log(35)} \]

6d) \( \log\left(\frac{x-3}{2}\right) + \log\left(\frac{x+2}{7}\right) = 0 \)
\[= \log\left[ \frac{(x-3)(x+2)}{14} \right] = 0 \]
\[\Rightarrow 10^0 = 1 = \frac{(x-3)(x+2)}{14} \]
\[\Rightarrow 14 = (x-3)(x+2) \]
\[\Rightarrow 14 = x^2 + 2x - 3x - 6 \]
\[\Rightarrow 0 = x^2 - x - 20 \]
\[= (x+4)(x-5) \]
\[= x+4 = 0 \quad \text{or} \quad x-5 = 0 \]
\[x = -4 \quad \text{or} \quad x = 5 \]

Throw the negative answer away.
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7) \( A = 3.6e^{rt} \)

At \( t=0 \): \( A_0 = 3.6e^{r(0)} = 3.6 \)

At \( t=10,000 \): \( A_{10,000} = 3.6e^{(-0.21 \cdot 10^{-4}) \cdot 10^4} = 3.6e^{-1.21} \)

8) \( A = P\left(1 + \frac{r}{n}\right)^{nt} \), \( P = 20,000 \), \( r = 0.06 \)

\[ n = 365, \quad t = 3 \]

\[ A = 20,000\left(1 + \frac{0.6}{365}\right)^{365(3)} \]

\[ \text{ROI} = \frac{20,000\left(1 + \frac{0.6}{365}\right)^{365(3)} - 20,000}{20,000} \]

\[ = \frac{20,000\left[(1 + \frac{0.6}{365})^{1095} - 1\right]}{20,000} = 0.197 \]

ROI = return on investment

9) \( A = Pe^{rt} = xe^{0.1t} \), \( x \neq 0 \)

\[ \Rightarrow 2x = xe^{0.1t} \]

\[ \Rightarrow 2 = e^{0.1t} \]

\[ \Rightarrow t = 10 \ln(2) \]