Solving Logarithmic Equations

- **one-to-one property:** a single logarithm on each side of the "=" same base.

ex $\log_3(x) = \log_3(4)$

$x = 4$

ex $\log_2(x) = \log_2(-7)$

$x = -7$ extraneous
no solution

- because log has a constraint.
  Domain of log $(0, \infty)$

Ex $\log(x-2) + \log(x+1) = \log(2x+2)$

$\log((x-2)(x+1)) = \log(2x+2)$

$(x-2)(x+1) = (2x+2)$

$x^2 + x - 2x - 2 = 2x + 2$

$x^2 - x - 2 = 2x + 2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ $x = -1$ extraneous

$x = 4$

$\log(4-2) + \log(4+1) = \log(2(4)+2)$

(+) (+) (+) ✓

$x = -1$

$\log(-1-2)$ ✗

(−)

Exponential Method - in case you can't do the one to one

random # outside of my logarithms.

ex $\log_2(x-3) - 3 = 0$

$+3 \quad +3$

$\log_2(x-3) = 3$

$2^3 = x-3$

$8 = x-3$

$11 = x$ ✓

$\log_2(11-3) = \log_2(8)$ ✓ Good solution

ex $2\log(x) - \log(4) = \log(2)$

$\log(x^2) - \log(4) = \log(2)$

$\log\left(\frac{x^2}{4}\right) = \log(2)$

$\frac{x^2}{4} = 2$

$x^2 = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

$\log(2\sqrt{2})$ ✓

$\log(-2\sqrt{2})$ ✗

$x = 2\sqrt{2}$

Check solutions $2\sqrt{2}, -2\sqrt{2}$