2.3 Zeroes of Polynomials

3-5-19
Math 2311
Palmer PreCal
Pg 1

in factored form

$f(x) = 3(x-7)^2(x+5)^4(2x-1)$

$x = 7, -5, \frac{1}{2}$

Linear factor theorem

Expand (Product of linear factors)

$f(x) = 3(x-7)(x-7)(x+5)(x+5)(x+5)(x+5)(2x-1)$

real zero: x-intercept

non real zero: not x-intercept

ex 1

$f(x) = x^6 + 7x^5 - 4x^2 + 3x - 2$

6^{th} degree polynomial - 6 solutions in total, could be a mix of both real and non real.

Ex2

$f(x) = x^3 + 2x^2 + 4x + 8$

$= (x^3 + 2x^2) + (4x + 8)$

GCF

$= x^2(x+2) + 4(x+2)$

$= (x^2+4)(x+2)$

Find Zeroes

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$\boxed{x = \pm i2}$

2 nonreal sol.

$x + 2 = 0$

$\boxed{x = -2}$

1 real sol.

Complex Conjugate theorem:

non real (complex) zeros appear in pairs.

$3 - 5i$

Complex Conjugate is $3 + 5i$ must also be a zero f(x) is a 9th degree polynomial w/ following zeros: $\sqrt{5}$, $7i$, 3

① Find another zero of f(x): $\underline{-7i}$

② What is the maximum # of non real zeroes f(x) has?

9th degree - I have - $7i, -7i, \sqrt{5}, 3$, ?, ?, ?, ?, ?

It could have up to 6. because it has $7i, -7i$, and 4 more because it comes in pairs.

③ What is the maximum # of real zeros f(x) has?

$\boxed{7}$ because we have $3, \sqrt{5}$, ?, ?, ?, ?, ?

Solving Polynomials Methods:

① factor by grouping ② Possible $\underbrace{\text{rational}}_{\text{fraction}}$ roots

$\boxed{ex}$ $\boxed{3}x^4 - 5x^3 + 2x \boxed{-7}$ * factors of the constant "p" and the leading coefficient "q"

$\pm \frac{p}{q}$

P: 1, 7

q: 1, 3

$\pm\frac{p}{q} = \pm\frac{1}{1}, \pm\frac{1}{3}, \pm\frac{7}{1}, \pm\frac{7}{3}$

$= \boxed{+1, -1, +\frac{1}{3}, -\frac{1}{3}, +7, -7, +\frac{7}{3}, -\frac{7}{3}}$

List the possible rational roots:

$F(x) = 6x^5 - 4x^3 + 2x + 4$

P: 4, 1, 2, 2 $\quad \pm\frac{p}{q} \rightarrow \pm\frac{4}{1}, \pm\frac{4}{6}, \pm\frac{4}{2}, \pm\frac{4}{3}$

q: 6, 1, 2, 3

$\pm\frac{1}{6}, \pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{3}$

$\pm\frac{2}{6}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{2}{3}$

$g(x) = 7x^3 + 2x^2 - 9x - 12$

P: (1, 12)

q: (1, 7)

$\pm\frac{p}{q} \rightarrow \pm\frac{1}{1}, \pm\frac{1}{7}, \pm\frac{12}{1}, \pm\frac{12}{7}$