2.3 Zeros of Polynomials

in factored form
\[ f(x) = 3(x-7)^2(3x^5-4x^2+3x-2) \]
\[ x = 7, -5, \frac{1}{2} \]

**Linear Factor Theorem**
Expand \( (Product \ of \ linear \ factors) \)
\[ f(x) = 3(x-7)(x-7)(3x^5+5)(3x^5+5)(3x^5+5) \]

real zero: x-intercept
non real zero: not x-intercept

6th degree polynomial - 6 solutions in total, could be a mix of both real and non-real.

**Ex. 2**
\[ f(x) = x^3 + 2x^2 + 4x + 8 \]
\[ = \left( x^3 + 2x^2 \right) + \left( 4x + 8 \right) \]
\[ = x^2(x+2) + 4(x+2) \]
\[ = (x^2+4)(x+2) \]

Find Zeros
\[ x^2 + 4 = 0 \quad \Rightarrow \quad x = \pm 2i \]
\[ x+2 = 0 \quad \Rightarrow \quad x = -2 \]
1 real sol.
2 non-real sol.

**Complex Conjugate Theorem:**
non real (complex) zeros appear in pairs.

**3 - 5i**

Complex Conjugate is \( 3 + 5i \)

must also be a zero
f(x) is a 9th degree polynomial with following zeros: \( \sqrt{5}, 1, 7i, 3 \)

1) Find another zero of \( f(x) \): \(-7i\)

2) What is the maximum \# of non real zeros \( f(x) \) has?
   9th degree - I have - 7i, -7i, \( \sqrt{5}, i, \frac{3}{2}, \frac{2}{3}, \frac{-3}{2}, \frac{-2}{3}, \)

   It could have up to 6 because it has 7i, -7i, and 4 more because it comes in pairs.

3) What is the maximum \# of real zeros \( f(x) \) has?

\[ \pm 7 \] because we have 3, \( \sqrt{5}, \frac{1}{2}, \frac{2}{3}, \frac{-3}{2}, \frac{-2}{3} \]

Solving Polynomials Methods:

1) factor by grouping
2) Possible rational roots

**Example**

\[ 3x^4 - 6x^3 + 2x - 7 \]

Possible rational roots: 
\[ \pm \frac{p}{q} \]

**Factors of the constant and the leading coefficient**

- \( p \)
- \( q \)

Factors: +1, -1, +1/3, -1/3, +7i, -7i, +7/3, -7/3

List the possible rational roots:

- \( f(x) = 6x^5 - 4x^3 + 2x + 4 \)
- \( g(x) = 7x^3 + 2x^2 - 9x - 12 \)

Possible rational roots:

- \( \pm \frac{p}{q} \) for \( p \) and \( q \)
  - \( p \) and \( q \) for \( 11, 12, 2, 11, 17, 12, 11, 17 \)