Graphing Rational functions

Math 2311 Mrs. Palmer
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$\boxed{ex}$ $F(x) = \dfrac{3x^2-12x+9}{x^2+x-12}$

Step 1: Factor top and bottom

$$\frac{3x^2-12x+9}{x^2+x-12} \rightarrow \frac{3(x^2-4x+3)}{(x-3)(x+4)} \rightarrow \frac{3(x-1)\cancel{(x-3)}}{\cancel{(x-3)}(x+4)}$$

Notice $(x-3)$ cancels out means there is a hole.

when $\boxed{x=3 \text{ there is a hole}}$

$F(x) = \dfrac{3(x-1)}{(x+4)}$ To find the y coordinate of the hole. Plug $x=3$ into the equation.

$$\frac{3(3-1)}{(3+4)} = \frac{3(2)}{7} \overset{\text{simplified}}{=} \frac{6}{7} \quad \boxed{(3, 6/7)}$$

Step 2: Vertical Asymptote - set the denominator equal to zero and solve.

$F(x) = \dfrac{3(x-1)}{(x+4)}$ $x+4=0$
$x \neq -4$

at $\boxed{x=-4 \text{ you have a V.A.}}$

Remebering on how to solve Rational equations:

$\boxed{ex}$ $\dfrac{6}{v-5} + \dfrac{v}{v-3} = \dfrac{24}{v^2-8v+15}$

$v^2-8v+15 \rightarrow (v-3)(v-5)$

$$(v-5)(v-3)\left(\frac{6}{v-5} + \frac{v}{v-3} = \frac{24}{(v-3)(v-5)}\right)$$

$6(v-3)+v(v-5)=24$

$6v-18+v^2-5v=24$

$v^2+v-18=24$

$v^2+v-42=0$

$(v-6)(v+7)=0$

$\boxed{v=6,-7}$

$v \neq 5, 3$

Step 3: Horizontal Asymptote

- when the highest exponent on top and bottom are the same then H.A. is the leading coefficient

$F(x) = \dfrac{3(x-1)}{(x+4)} \rightarrow \dfrac{3x^1-3}{x^1+4}$

highest exponent is 1 on both top and bottom.
the H.A. is $\frac{3}{1} = 3$ $\boxed{y=3 \text{ H.A.}}$

Step 4: y-intercept + x-intercept

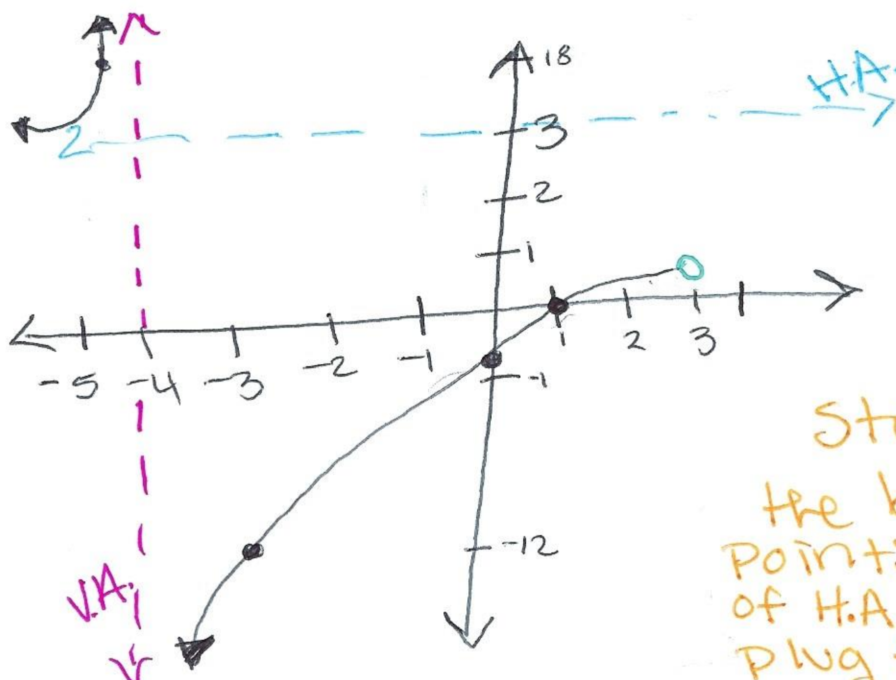
$\boxed{\text{y-int}}$ $F(x) = \dfrac{3x-3}{x+4} \rightarrow y = \dfrac{3(0)-3}{0+4} = \dfrac{-3}{4}$ $\boxed{\left(0, \frac{-3}{4}\right)}$

$\boxed{\text{x-int}}$

$\dfrac{3x-3}{x+4} = 0 \rightarrow x+4\left(\dfrac{3x-3}{x+4} = 0\right) \rightarrow 3x-3=0$

$3x = 3 \rightarrow x = 1$ $\boxed{(1,0)}$

Graph



Step 6: to test the behavior take points on either side of H.A. and V.A. and plug them in the function.

Take $x = -3$ $f(x) = \dfrac{3(-3)-3}{-3+4} = \dfrac{-9-3}{1} = \dfrac{-12}{1} = -12$ $(3,-12)$

Take $x = -5$ $f(x) = \dfrac{3(-5)-3}{-5+4} = \dfrac{-15-3}{-1} = \dfrac{-18}{-1} = 18$ $(-5,-18)$

More on H.A.

- when the top exponent is smaller and the bottom exponent is bigger

  HA is $y=0$ $\boxed{ex}$ $f(x) = \dfrac{x^{1}+1}{2x^{2}-2x-12}$

- when the top exponent is bigger and the bottom exponent is smaller

  No H.A. $\boxed{ex}$ $f(x) = \dfrac{x^{2}-3x-10}{x^{1}+3}$

  but you get a <u>Slant A.</u>

  * To find S.A. you divide by using <u>long division</u> or <u>synthetic division</u>.

$$
\begin{array}{r}
x - 6 \\
x+3 \,\overline{\smash{)}\, \cancel{x^2} - 3x - 10} \\
-(\cancel{x^2} + 3x) \\
\hline
-\cancel{6x} - 10 \\
-(\cancel{-6x} - 18) \\
\hline
8
\end{array}
$$

our slant A. is $y = x - 6$
we don't need the remainder.