Mutually Exclusive: e.g. Making A in Contemporary math and making a B.

Independent: e.g. Tossing a die and flipping a coin.

- The occurrence of one has no effect on the probability of the other.

\[ P(\text{Faucard}) = \frac{12}{52} = 0.23. \]

\[ P(\text{any certain event}) = 1 \]
\[ P(\text{impossible event}) = 0. \]

\[ P(C^c) = 1 - P(C) \]
\[ P(C^c) = 1 - P(C^c). \]
Specific Thoughts: Sample spaces, outcomes, relating the computation of theoretical probability to sets, properties of theoretical probability (listed below), computing probability using complements, and computing empirical probability.

Key Terms:

Probability Experiment – process that leads to well defined results called outcomes

Outcome – the result of a single trial of a probability experiment

Sample Space – the set of all possible outcomes of a probability experiment

Event – any subset of the sample space of a probability experiment

Mutually Exclusive – two events are mutually exclusive when they cannot occur at the same time.

Independent – two events, A and B, are independent if A occurring has no effect on the probability of B occurring.

Complement – If A is an event, then the complement of A, \(A^c\), is the event where A does not occur.

Theoretical Probability – assumes all outcomes are equally likely and determines the probability of an event occurring without performing experiments.

Empirical Probability – based on observed frequency, meaning the number of times an event occurs out of a certain number of trials.

- Sample Spaces, Outcomes, and Events
  
  Example – In rolling a single die, the die could land with 1, 2, 3, 4, 5, or 6 face up. These are all the possible results or outcomes of rolling the die, so the sample space is the set
  
  \[ U = \{1, 2, 3, 4, 5, 6\} \]

  Rolling an even number is an event that could occur as a result of rolling the die. If E is the event we roll an even number, then
  
  \[ E = \{2, 4, 6\} \]

  Example – In flipping two coins, each coin could either land on heads or tails. The sample space is then
  
  \[ U = \{HH, HT, TH, TT\} \]

  where H represents a coin landing heads up and T represents a coin landing tails up.

  Getting at least one tail is an event that could occur as a result of flipping the two coins. If R is the event we get at least one tail, then

  \[ R = \{HT, TH, TT\} \]

  The sum of the probabilities of the outcomes in the sample space is 1, so \( P(U) = 1 \).
• **Theoretical Probability** – Assuming all outcomes have equal likelihood of occurring, the theoretical probability of event A occurring is

\[ P(A) = \frac{n(A)}{n(U)} \]

where U is the sample space and \( n(A) \) is the cardinality of the set A.

**Note:** The notation for probability is the same as the notation for a power set. The context of the problem should allow you to determine which is being referred to.

- Example – In rolling a single die, the sample space is \( U = \{1, 2, 3, 4, 5, 6\} \) and \( n(U) = 6 \). If the event E is rolling an even number, then \( E = \{2, 4, 6\} \) and \( n(E) = 3 \). The probability that event E occurs is then

\[ P(E) = \frac{n(E)}{n(U)} = \frac{3}{6} = \frac{1}{2} \]

- Example – Determine the probability of drawing a 7 from a standard deck of 52 cards.

Solution – The sample space, U, for the standard deck of 52 cards is

\[
\begin{array}{cccccccccccc}
\text{A} \spadesuit & \text{2} \spadesuit & \text{3} \spadesuit & \text{4} \spadesuit & \text{5} \spadesuit & \text{6} \spadesuit & \text{7} \spadesuit & \text{8} \spadesuit & \text{9} \spadesuit & \text{10} \spadesuit & \text{J} \spadesuit & \text{Q} \spadesuit & \text{K} \spadesuit \\
\text{A} \heartsuit & \text{2} \heartsuit & \text{3} \heartsuit & \text{4} \heartsuit & \text{5} \heartsuit & \text{6} \heartsuit & \text{7} \heartsuit & \text{8} \heartsuit & \text{9} \heartsuit & \text{10} \heartsuit & \text{J} \heartsuit & \text{Q} \heartsuit & \text{K} \heartsuit \\
\text{A} \diamondsuit & \text{2} \diamondsuit & \text{3} \diamondsuit & \text{4} \diamondsuit & \text{5} \diamondsuit & \text{6} \diamondsuit & \text{7} \diamondsuit & \text{8} \diamondsuit & \text{9} \diamondsuit & \text{10} \diamondsuit & \text{J} \diamondsuit & \text{Q} \diamondsuit & \text{K} \diamondsuit \\
\text{A} \clubsuit & \text{2} \clubsuit & \text{3} \clubsuit & \text{4} \clubsuit & \text{5} \clubsuit & \text{6} \clubsuit & \text{7} \clubsuit & \text{8} \clubsuit & \text{9} \clubsuit & \text{10} \clubsuit & \text{J} \clubsuit & \text{Q} \clubsuit & \text{K} \clubsuit \\
\end{array}
\]

The possible outcomes for the event of drawing a 7, D, are the elements of the subset

\[
\begin{array}{cccc}
\text{7} \spadesuit & \text{7} \heartsuit & \text{7} \diamondsuit & \text{7} \clubsuit \\
\end{array}
\]

So,

\[ P(D) = \frac{n(D)}{n(U)} = \frac{4}{52} = \frac{1}{13} \]

- **Complements** – If E is an event, then \( E^c \) is the event that E does not occur. Recall from set theory

\[ E \cup E^c = U \]

So,

\[ P(E) + P(E^c) = P(U) \]

Since \( P(U) = 1 \), we have \( P(E) + P(E^c) = 1 \). Rearranging this equation results in

\[ P(E^c) = 1 - P(E) \]

Which serves as a very useful tool in calculating the probability of the complement of an event.
Example – Of the 32 trials on the docket in a county court, 5 are homicides, 12 are drug offenses, 6 are assaults, and 9 are property crimes. If jurors are assigned cases randomly, what’s the probability a juror will not get a homicide case?

Solution – There are 32 trials and 5 are homicide, so the probability of a juror getting a homicide case is $\frac{5}{32}$. The probability of not getting a homicide is then

$$1 - \frac{5}{32} = \frac{27}{32}$$

- **Empirical Probability** – determining the probability of an event by observing the amount of times the event occurs in a set number of trials during an experiment. If $n$ trials are performed in an experiment and an event, $E$, occurs $f$ times, then

$$P(E) = \frac{f}{n}$$

- Theoretical probability is the “on paper this is what should happen” form of probability. Empirical probability is the “let’s get our hands dirty and really see what’s going to happen” form of probability. What’s neato is that as the number of trials performed in an experiment increases, the empirical probability of an event gets closer and closer to the theoretical probability of the event occurring. This will be demonstrated in a group activity.

Example – In a random survey of 500 people, 210 had type O blood, 223 had type A blood, 51 had type B blood, and 16 had type AB. What is the probability a randomly selected person from the general population has:

- type O blood?

Solution: Each of the 500 people represents a trial and there were 210 occurrences of type O blood so

$$P(\text{having type O blood}) = \frac{210}{500} \approx 0.42$$

- type A or type AB blood?

Solution: 223 had type A and 16 had type AB, therefore there were $223 + 16 = 239$ occurrences of either A or AB. So, we have

$$P(\text{having type A or AB blood}) = \frac{239}{500} \approx 0.478$$

Notice a person cannot have both type A and type AB blood so the events are mutually exclusive. When events A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$

- a blood type other than type B?

Solution: We can find the probability of not having type B blood by subtracting the probability of having type B blood from 1.

$$P(\text{not having type B blood}) = 1 - \frac{51}{500} = \frac{449}{500} = 0.898$$
Module #3
Worksheet and Practice Problems #3 – Sample Spaces and Theoretical Probability

Spinning a Spinner

1. Imagine spinning the spinner above one time. Determine the sample space for the spinner.
   \[ S = \{ \text{white, gray, black} \} \]

2. If you spun the spinner one time, what is the probability it would land on a grey piece?
   \[ \frac{2}{4} \]

3. If you spun the spinner one time, what is the probability it would land on a white piece?
   \[ \frac{3}{4} \]

4. If you spun the spinner one time, what is the probability it would land on a black piece?
   \[ \frac{1}{4} \]

5. If you spun the spinner one time, what is the probability it would land on either a white piece or a black piece?
   \[ \frac{3}{4} + \frac{1}{4} = \frac{5}{4} \]

Rolling a die

6. Imagine rolling the die above one time. Determine the sample space for the die.
   \[ S = \{1, 2, 3, 4, 5, 6\} \]

7. If you rolled the die one time, what is the probability it would land on an odd number?
   \[ \frac{3}{6} = \frac{1}{2} \]

8. If you rolled the die one time, what is the probability it would land on a number less than 5?
   \[ \frac{4}{6} = \frac{2}{3} \]

9. If you rolled the die one time, what is the probability it would land on 7? Explain your reasoning.
   \[ 0 = \text{Impossible event.} \]

10. If you rolled the die one time, what is the probability it would land on 1, 2, 3, 4, 5 or 6? Explain.
    \[ 1 = \text{Certain event.} \]
11. If the spinner is spun once and the die is rolled once, what is the probability the spinner will land on white and the die will land on a number greater than 3?

\[ \frac{4}{6} \times \frac{3}{4} = \frac{1}{2} \times \frac{1}{2} = \frac{4}{14} \]

Multiply

12. If the spinner lands on black instead of white, will it effect the outcome of the die?

No - Because they are disjoint events

13. If the spinner is spun once and the die is rolled once, what is the probability the spinner will land on a color other than grey and the die will land on a number less than or equal to 2?

\[ \frac{5}{4} \times \frac{2}{6} = \frac{5}{7} \times \frac{1}{3} = \frac{5}{21} \]

(14 – 20) Let event A be the event of rolling a single die once and getting an even number. Let event B be the event of rolling a single die once and getting a prime number.

Prime: \{2, 3, 5\} = Divisible by 1 and itself except 1

14. Determine the set of outcomes for each event.

15. Find \( P(A) \) = \( \frac{1}{2} \)

16. Find \( P(B) \) = \( \frac{1}{2} \)

17. Find \( P(A \cup B) \) = \( \frac{1}{2} + \frac{1}{2} = P(\text{even or prime}) = \frac{5}{6} \)

HB: even or prime = \{2, 3, 4, 5, 6\}

18. Is \( P(A \cup B) = P(A) + P(B) \)? What could be an explanation for your answer?

\[ \frac{5}{6} \neq \frac{1}{2} \]

Reason: They are independent events

19. Find \( P(A \cap B) \) = \( \frac{2}{6} = \frac{1}{3} \)

20. Is \( P(A \cap B) = P(A) \cdot P(B) \)? What could be an explanation for your answer?

\[ \frac{1}{3} \neq \frac{1}{2} \cdot \frac{1}{2} \Rightarrow \frac{1}{3} \neq \frac{1}{4} \]

Dependent events