INSTRUCTOR: MR BRICE
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Module #3

Practice Problems #1 – Counting and Permutations (all/some objects)

1. Evaluate the following:
   a. $\frac{13!}{9!} = \frac{13\times12\times11\times10\times9!}{9!} = 13\times12\times11\times10$
   b. ${}_7P_2$
   c. $\frac{11!}{(11-4)!}$
   d. ${}_5P_0 = \frac{5!}{(5-0)!} = 1$

2. Describe the difference between finding the number of ways to choose five objects if each can only be chosen once and the number of ways if the objects can be chosen more than once.

3. How many four-letter passwords can be formed from the word *shifter* if each letter must be distinct?
   ~~Ord~~ order Matters (Permutation).
   ~~7~~$P_4 = \frac{~~7~~!}{(~~7~~-4)!} = \frac{~~8~~7\times6\times5\times4\times~~3~~2!}{~~3~~2!} = ~~8~~7\times6\times5\times4$

4. How many four-letter passwords can be formed from the word *shifter* if letters can be repeated?
   ~~Order does not~~ Order matters.
   Fundamental Counting principle with repetition.
   ~~8 · 8 · 8 · 8~~ 7 · 7 · 7 · 7.

5. In Texas, license plates have three letters followed by four numbers. How many license plates can be formed?
   26 · 26 · 26 · 10 · 10 · 10 · 10 ~~or~~ ~~Permutation~~ · ~~Numbers~~
   (letters) (#)
   = 175,760,000.

6. Out of 19 contacts in your cell phone, how many ways can you set the first three speed-dial contacts?
   Assume order does not matter: Combination.
   $19C_3 = \frac{19!}{(19-3)!\,3!}$

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Worksheet #2 – Permutations with like objects and Combinations

1. There are 3 girls and 5 boys on the swim team. How many ways can they be arranged for a photo?

Order matters: Permutation ; with like objects.

k_1 (Guys) $= 5$, Total $= 8$

k_2 (Ladies) $= 3$

$$= \frac{8!}{5! \times 3!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{3! \cdot \cancel{5!}} = \frac{8 \times 7 \times 6}{3 \times 2}$$

2. In how many ways can you rearrange the letters in *critter*?

$$\frac{7!}{1!\,2!\,2!\,1!\,1!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!} \cdot 2!}$$

3. In 7-card poker, each player is dealt 7 cards from a standard deck of 52 cards. How many different hands can be dealt?

$_{52}C_7$ or ~~52 51 50 49 48 47 46~~

$$\frac{52!}{(52-7)!\,7!} =$$

4. A company is looking to hire 4 people for identical positions. If there are 12 men and 9 women being interviewed, in how many ways can 2 men and 2 women be chosen?

$$_{12}C_2 \cdot {}_{9}C_2 = \frac{12!}{(12-2)!\,2!} \cdot \frac{9!}{(9-2)!\,2!}$$

5. In how many ways can at least 3 women be chosen?

6. You have 5 different pizza toppings to choose from.
   a. How many 0-topping pizzas can you make? $_5C_0$
   b. How many 1-topping pizzas can you make? $_5C_1$
   c. How many 2-topping pizzas can you make? $_5C_2$
   d. How many 3-topping pizzas can you make? $_5C_3$
   e. How many 4-topping pizzas can you make? $_5C_4$
   f. How many 5-topping pizzas can you make? $_5C_5 = \frac{5!}{(5-5)!\,5!} = 1$

Module #3

Practice Problems #2 – Permutations with like objects and Combinations

1. Decide if each selection is a permutation or a combination:

   a. A 7-digit passcode is chosen from the numbers 1 through 9.

   b. A bride chooses 3 types of cookies from 12 to serve at her reception. Combination.

   c. 5 rowdy students are chosen out of 30 for lunch cleanup duty. Combination.

   d. The number of ways a president a vice president can be chosen from 11 candidates. Permutation.

2. In how many ways can 3 red marbles, 7 blue marbles, and 5 green marbles be arranged?

   ~~3·7·5.~~ Permutation with like objects.

   $r-3 \quad g-5 \quad n=15 \cdot \quad \frac{15!}{3!5!7!}$

   $b-7$

3. A Powerball ticket has you pick five numbers out of the numbers from 1 to 69 and one special number (the powerball) from the numbers 1 to 26. How many Powerball tickets are possible?

   $69C_5 \cdot 26 \cdot = \frac{69!}{(69-5)!5!} \cdot 26 \cdot$

4. Out of all those Powerball tickets, for each drawing, how many sets of numbers are winners?

5. In how many ways can a gardener pick 4 vegetable plants for his garden from 10 choices?

   $10C_4 = \frac{10!}{(10-4)!4!}$

6. A four-person crew for the international space station is to be chosen from a candidate pool of 10 American astronauts and 12 Russian astronauts. How many different crews are possible if there must be 2 Americans and 2 Russians?

   $10C_2 \cdot 12C_2 = \frac{10!}{(10-2)!2!} \cdot \frac{12!}{(12-2)!2!}$

7. How many different crews are possible if there must be 3 Americans and 1 Russian?

   $10C_3 \cdot 12C_1$

8. How many different crews are possible if all four members are to be American?

   $10C_4 \cdot 12C_0$

9. How many different crews are possible if there must be at least 2 Americans?

   $(10C_2 \cdot 12C_2) + (10C_3 \cdot 12C_1) + (10C_4 \cdot 12C_0)$