(e) Restaurants that make a great sandwich ⇒ Not a set b'cos it is subjective.

2. How are sets similar to statements in logic?

3. Give three different ways of describing the empty set.

4. Is it true that every element of the natural numbers is an element of the integers? Why or why not? $\mathbb{N} - \{1,2,3,4 \ldots\}$. $\mathbb{Z} - \{\ldots -3,-2,-1,0,1,2,3 \ldots\}$ TRUE $\mathbb{N} \in \mathbb{Z}$.

5. Is it true that every element of the integers is an element of the natural numbers? Why or why not? No. $\{-3\} \notin \mathbb{N}$.

6. If you are discussing the natural numbers, what is the universal set? Why? $\mathbb{R}$ or $\mathbb{Z}$.

7. Let $A = \{1, 2, 3\}$. Is $\emptyset \in A$? False.

Definition Let A and B be sets. A is a subset of B if and only if every element of A is also an element of B. The notation here is $A \subseteq B$.

8. Argue that each of the following ~~are~~ is true:

(a) For any set A, $\emptyset \subseteq A$ TRUE: The empty set is a subset of every set.

(b) For any set A, $A \subseteq A$ TRUE: Every element of A is also in A.

9. Let A, B, and C be sets. Argue that if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
Explanation in notes.

Definition Let A and B be sets. Then A and B are equal sets (notation is $A = B$) if and only if $A \subseteq B$ and $B \subseteq A$. → Explanation in notes.

Definition Let A and B be sets. A is a proper subset of B if and only if B contains elements that are not elements of A.The notation here is $A \subset B$. → Explanation in notes.

10. Give an example of two sets so that $A \subset B$. → In notes.

11. Argue that if A and B are sets with no elements, then $A = B$. $\{\} = \phi$

12. Answer each of the following, giving justification for your responses.

(a) Is $\emptyset \subseteq \emptyset$? Yes: Every set is a subset of itself.

(b) Is $\emptyset \subset \emptyset$? No: Nothing left over.

(c) Is $\emptyset \subseteq \{\emptyset\}$? Yes: Empty set is a subset of every set.

(d) Is $\emptyset \in \{\emptyset\}$? Yes: the only element in $\{\phi\}$ is ϕ.

(e) Is $\emptyset \subset \{\emptyset\}$?

(f) Is $\emptyset \in \emptyset$? No

(P2)

Is there an element in A that is not in B?

(1) If no, $A \subseteq B$.

(2) If yes, A is not a subset of B. ie $A \not\subseteq B$

Eg: $A = \{1, 2, 3\}$.

$B = \{1, 2, 3, 4\}$.

$\Rightarrow A \subseteq B$.

$B \not\subseteq A$.

$\mathbb{N} \subseteq \mathbb{Z}$.

T.

$\mathbb{Z} \subseteq \mathbb{N}$. $\Rightarrow$ false.

Reason: There exists an element in $\mathbb{Z}$ (-3) that does not belong to $\mathbb{N}$.

$A \subseteq A \Rightarrow$. Every set is a subset of itself.

Explanation to question 9

Given: Sets A, B, C.

$A \subseteq B$; $B \subseteq C$

Prove: $A \subseteq BC$

Soln:

① Choose an element in A, n_1.

② $n_1 \in B$; Def. of sub.

③ $n_1 \in C$; " " "

④ $A \subseteq C$; Every element in A is in C.

Equal sets:

①– Even numbers between 1 and 11.
ie $\{2, 4, 6, 8, 10\}$.

②– Positive numbers Divisible by through 10.

Note that $1 = 2$.

Also: $\mathbb{N} = \text{Pos}\,\mathbb{Z}$.

$\mathbb{N}$ → natural nos

$\text{Pos}\,\mathbb{Z}$ → positive integers.

Proper subset:

$A = \{1, 2, 3\}$.

$B = \{1, 2, 3, 4\}$.

Is $A \subseteq B$? ✓

Is $A \subset B$? ✓

Why: The reason is because B contains at least ~~an element~~ one element that does not belong to A.