

Jabberwocky and the Transitive Property of Logic

Many of you are familiar with the Transitive Property of Equality as listed in many mathematics textbooks. Simply stated, it says: If $a = b$ and $b = c$, then $a = c$. But there are also transitive properties that work in the realm of logic, that apply to relations other than equality. For example, "If Mary IS OLDER THAN Joe, and Joe IS OLDER THAN Daisy, then Mary IS OLDER THAN Daisy."

1) Is the relation "Is Older Than" one for which the Transitive Property always holds? If you say "no", give an example.

2) Think of another relation that you believe holds for the Transitive Property; write an example of the relation.

faster,
is stronger than, weighs more than,
Smarter (IQ).

3) Think of a relation for which the Transitive Property does NOT hold; write a. the example and explain why it is not logical.

① Tom is the father of Bill; Bill is the father of Joe, But Tom is NOT the father of Joe. ② Is similar to ③ Is friends with

Relations may be Transitive, Intransitive or Nontransitive. Transitive we know; here are explanations for the other two.

Intransitive Relations – these hold for a and b and for b and c, but it does not hold for a and c FOR ANY a, b and c.

See if you can come up with an Intransitive Relation; remember the For Any part of the definition. "Is the father of"

Nontransitive Relations – these hold for a and b and for b and c, but MAY OR MAY NOT hold for a and c, depending on the a, b and c substituted.

See if you can come up with a Nontransitive Relation, one that holds sometimes but not always.

"Is friends with"

So why "Jabberwocky" in the title? Some of you may recognize the word as the title to the nonsensical poem by Lewis Carroll who was not only the author of the Alice books, but who was an esteemed mathematician and logician at Oxford. As an amusement, Carroll created the following logical puzzles. The object of each is to string the assertions so that a relates to b, b relates to c, ... e relates to f, until you can summarize in one logical statement relating a and f. Consider this one.

- (a) All babies are illogical.
- (b) Nobody is despised who can manage a crocodile.
- (c) Illogical persons are despised.

Babies can't manage a crocodile.

There are four concepts in the statements: babies, being logical, managing crocodiles and being despised. Your task is to use the information in all the statements to come to a final conclusion, a conclusion which mentions only two of the concepts. See what you can do.

Babies can't manage a crocodile.

All right, try these!

1) My saucepans are the only things I have that are made of tin.

I find all of your presents very useful.

Presents \leftrightarrow Useful \leftrightarrow saucepan \leftrightarrow Tin

None of my saucepans are of the slightest use.

Summary conclusion: Presents are not made of tin.

2) No potatoes of mine, that are new, have been boiled.

All my potatoes in this dish are fit to eat.

No unboiled potatoes of mine are fit to eat.

Summary conclusion: No potato in this dish is fit to eat.

3) No ducks waltz.

No officers ever decline to waltz.

All my poultry are ducks.

Summary conclusion: None of my poultry is an officer.

4) Everyone who is sane can do logic.

Sane \leftrightarrow logic

No lunatics are fit to serve on a jury.

(Insane \leftrightarrow Jury

None of your sons can do logic.

Sons \leftrightarrow logic.

Summary conclusion: None of your sons can serve on a jury.

5) No experienced person is incompetent.

experienced \rightarrow incompetent.

Jenkins is always blundering.

Jenkins \rightarrow blundering.

No competent person is always blundering.

competent \rightarrow blundering.

Summary conclusion: Jenkins is not experienced.

6) All puddings are nice.

This dish is a pudding.

No nice things are wholesome.

Summary conclusion: This dish is not wholesome.

7) No one takes The Times unless he is well educated.

No hedgehogs can read.

Those who cannot read are not well educated.

Summary conclusion:

8) Not for the faint of heart!

The only animals in this house are cats.

Every animal that loves to gaze at the moon is suitable for a pet.

When I detest an animal, I avoid it.

No animals are carnivorous, unless they prowl at night.

No cat fails to kill mice.

No animals ever take to me, except what are in this house.

Kangaroos are not suitable for pets.

None but carnivora kill mice.

I detest animals that do not take to me.

Animals, that prowl at night, always love to gaze at the moon.

Summary conclusion: