Solving Quadratic Equations

Linear Equations vs Quadratic Equations

3x + 2 = 23
Degree → 1
Solutions → 1

Degree → 2nd
Solutions → 2

Three Methods for solving Quadratic Equations

Method 1: The Factoring Method

\( x^2 + 5x + 6 = 0 \)

\((x + 2)(x + 3) = 0\)

1) Set to 0.

2) Factor the polynomial

The property of zero

\[ \text{If } a \cdot b = 0 \text{ then } a=0 \text{ or } b=0 \]

3) Set each factor equal to 0 and solve.

Ex 1: \( 28x^2 + 4x = 0 \)

\( 4x(7x + 1) = 0 \)

\( 4x = 0 \) or \( 7x + 1 = 0 \)

\( x = 0 \) or \( \frac{-1}{7} \)

Solutions: \( x = 0 \) and \( x = -\frac{1}{7} \)
Ex 2: \[36x^2 - 1 = 0\]
\[ (6x - 1)(6x + 1) = 0 \]
\[
\begin{align*}
6x - 1 &= 0 \\
+1 &
\end{align*}
\]
\[
\begin{align*}
6x &= 1 \\
\frac{6x}{6} &= \frac{1}{6} \\
x &= \frac{1}{6}
\end{align*}
\]
\[
\begin{align*}
6x + 1 &= 0 \\
-1 &
\end{align*}
\]
\[
\begin{align*}
6x &= -1 \\
\frac{6x}{6} &= \frac{-1}{6} \\
x &= -\frac{1}{6}
\end{align*}
\]
Solutions: \[x = \frac{1}{6}\] and \[x = -\frac{1}{6}\]

Ex 3: \[4x^2 - 28 = 0\]
\[ (2x - 7)(2x + 7) = 0 \]
\[
\begin{align*}
2x - 7 &= 0 \\
+7 &
\end{align*}
\]
\[
\begin{align*}
2x &= 7 \\
\frac{2x}{2} &= \frac{7}{2} \\
x &= \frac{7}{2}
\end{align*}
\]
\[
\begin{align*}
2x + 7 &= 0 \\
-7 &
\end{align*}
\]
\[
\begin{align*}
2x &= -7 \\
\frac{2x}{2} &= \frac{-7}{2} \\
x &= -\frac{7}{2}
\end{align*}
\]
Solutions: \[x = \frac{7}{2}\] and \[x = -\frac{7}{2}\]

Difference of Squares = 0
Solutions will be opposites of each other

Ex 4: \[x^2 - 3x + 2 = 0\]
\[ (x - 1)(x - 2) = 0 \]
\[
\begin{align*}
x - 1 &= 0 \\
+1 &
\end{align*}
\]
\[x = 1\]
\[
\begin{align*}
x - 2 &= 0 \\
+2 &
\end{align*}
\]
\[x = 2\]
Solutions: \[x = 1\] and \[x = 2\]
Ex 5:

\[ 81x^2 + 36x + 4 = 0 \]

Solutions: \( x = \frac{2}{9} \)

Perfect Square = 0

Solutions are identical

\[ (9x + 2)(9x + 2) = 0 \]

\[ 9x + 2 = 0 \quad \text{or} \quad 9x + 2 = 0 \]

\[ \frac{-2}{9} \quad \frac{-2}{9} \]

\[ x = \frac{-2}{9} \quad x = \frac{-2}{9} \]

Ex 6:

\[ 2x^2 = 7x + 15 \]

Solutions: \( x = 5 \)

and

\[ x = -\frac{3}{2} \]

\[ \frac{-7x}{2} \quad \frac{-7x}{2} \]

\[ 2x^2 - 7x = 15 \]

\[ -15 \quad -15 \]

\[ 2x^2 - 7x - 15 = 0 \]

\[ (2x + 3)(x - 5) = 0 \]

\[ 2x + 3 = 0 \quad \text{or} \quad x - 5 = 0 \]

\[ \frac{-3}{2} \quad 5 \quad +5 \]

\[ 2x = \frac{-3}{2} \quad x = 5 \]

\[ x = \frac{-3}{2} \]
Ex 2:

\[
\frac{1}{12} x^2 - x - 24 = 0
\]

\[
x^2 - 12x - (12 \cdot 24) = 12 \cdot 0
\]

\[
x^2 - 12x - 288 = 0
\]

\[
(x - 24)(x + 12) = 0
\]

\[
x - 24 = 0 \text{ or } x + 12 = 0
\]

\[
x = 24 \quad x = -12
\]

Solutions: 
\[x = -12\]

\[x = 24\]