

3/4

1.5 Rational Expressions

Ratio \rightarrow (Fractional)

Ex:
$$\frac{15x^2}{25x} = \frac{\cancel{5} \cdot 3 \cdot \cancel{x} \cdot x}{5 \cdot \cancel{5} \cdot x} = \frac{3x}{5}$$

Ex:
$$\frac{18x^3y^2}{60x^2y^5} = \frac{\cancel{2} \cdot 3 \cdot \cancel{3} \cdot x \cdot \cancel{x} \cdot x \cdot \cancel{y} \cdot \cancel{y}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 5 \cdot x \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{y} \cdot \cancel{y} \cdot \cancel{y}} = \frac{3x}{10y^3}$$

Ex:
$$\frac{2x^2y}{yx-y} = \frac{\cancel{2} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{y}}{y(x-1)} = \frac{2x^2}{x-1}$$

Factor out
the biggest common
factor

Ex:
$$\frac{9x^2+9x}{2x+2} = \frac{\cancel{9}x(\cancel{x+1})}{2(\cancel{x+1})} = \frac{9x}{2}$$

Ex:
$$\frac{x^2+8x-20}{x^2+11x+10} = \frac{(\cancel{x+10})(x-2)}{(\cancel{x+10})(x+1)} = \frac{x-2}{x+1}$$

We cancel factors, things multiplied, not terms, things added or subtracted.

Ex: strange one

$$\frac{x-5}{10-2x} = \frac{x-5}{2(5-x)} = \frac{x-5}{-2(x-5)} = \frac{1}{-2}$$

$$x-5 \neq 5-x$$

$$x-5+5-x=0$$

so they are opposites

$$\begin{aligned} x-5 &= -(5-x) \\ &= -5+x \\ &= x-5 \end{aligned}$$

Also you can remember that $\frac{\text{any number}}{\text{its opposite}} = -1$

So you are free to cancel opposites if you introduce a factor of -1 .

Ex of opposites:

$$\frac{a-b}{b-a} = -1 \quad , \quad \frac{y-3}{3-y} = -1$$

$$\frac{a-b}{a+b} \neq -1 \quad \text{not opposites}$$

$$a-b+a+b = 2a \quad \text{not } 0$$

Ex:

$$\frac{2-x+2x^2-x^3}{x^2-4} = \frac{2+2x^2-x-x^3}{(x+2)(x-2)} = \frac{2(1+x^2)-x(1+x^2)}{(x+2)(x-2)}$$

factoring by grouping

$$= \frac{(2-x)(1+x^2)}{(x+2)(x-2)}$$

difference of 2 squares

$$= \frac{(2-x)(1+x^2)}{(x+2)(x-2)}$$

Cancelled opposites
So introduce a factor of -1

$$= \frac{-1 \cdot (1+x^2)}{(x+2)}$$

$$= -\frac{(1+x^2)}{(x+2)}$$

$$\begin{aligned} \text{Ex: } \frac{y^3 - 2y^2 - 3y}{y^3 + 1} &= \frac{y(y^2 - 2y - 3)}{(y+1)(y^2 - y + 1)} = \frac{y(y-3)\cancel{(y+1)}}{\cancel{(y+1)}(y^2 - y + 1)} \\ &= \frac{y(y-3)}{(y^2 - y + 1)} \end{aligned}$$

multiplication & Division:

$$\begin{aligned} \text{Ex: } \frac{x}{x-1} \cdot \frac{x^2-1}{x^2} &= \frac{x \cdot (x^2-1)}{(x-1) \cdot x^2} = \frac{x \cdot \cancel{(x-1)} \cdot (x+1)}{\cancel{(x-1)} \cdot x \cdot x} \\ &= \frac{(x+1)}{x} \end{aligned}$$

$$\begin{aligned} \text{Ex: } \frac{x^2 - x - 6}{x^2 + 6x + 9} \cdot \frac{x+3}{x^2-4} &= \frac{\cancel{(x-3)}(x+2) \cdot \cancel{(x+3)}}{(x+3)\cancel{(x+3)} \cdot (x-2)\cancel{(x+2)}} \\ &= \frac{(x+3)}{(x+3)(x-2)} \end{aligned}$$

opposite

$$\begin{aligned} \text{Ex: } \frac{4y-16}{5y+15} \div \frac{4-y}{2y+6} &= \frac{4y-16}{5y+15} \cdot \frac{2y+6}{4-y} = \frac{4(y-4) \cdot 2(y+3)}{5(y+3) \cdot \cancel{(4-y)}} \\ &= \frac{-1 \cdot 4 \cdot 2}{5} \\ &= -\frac{8}{5} \end{aligned}$$