3/1 Factoring Polynomials

IV. Factorable trinomials

Recall that $(x+3)(x+5) = x^2+3x+5x+15 = x^2+8x+15$

Now start with $x^2+8x+15$ and find its original factors.

Look for two numbers whose product is 15 and sum is 8.

These numbers are +5 and +3. So the two binomials are $(x+5)$ & $(x+3)$.

Ex. 1) $x^2+7x+12 = (x+3)(x+4)$

Thinking about factors

Product = 12	Sum = 7
5 · 4	3+4=7
	-3+-4=-7
-3 · -4	6+2=8
	-6+-2=-8
6 · 2	1+12=13
	-1+-12=-13
-2 · -6	
1 · 12	
-1 · -12	

Ex. 2) $x^2+9x+14 = (x+2)(x+7)$

Product = 14	Sum = 9
1 · 14	1+14=15
	-1+-14=-15
-1 · -14	2+7=9
2 · 7	
	-2+-7=-9
-2 · -7	

Ex 3) $x^2 - 12x + 20 = (x-2)(x-10)$

Product = 20	Sum = -12
$1 \cdot 20$	$1+20 = 21$
$-1 \cdot -20$	$-1+-20 = -21$
$-5 \cdot -4$	$-5+-4 = -9$
$5 \cdot 4$	$5+4 = 9$
$2 \cdot 10$	$2+10 = 10$
$-2 \cdot -10$	$-2+-10 = -12$

Notice: We can see from all the previous examples that if the 3rd term in the trinomial then the signs of the binomials will match.

Ex 4) $x^2 - 2x - 35 = (x-7)(x+5)$

Product = -35	Sum = -2
$-7 \cdot 5$	$-7+5 = -2$
$-5 \cdot 7$	$-5+7 = 2$
$-1 \cdot 35$	$-1+35 = 34$
$-35 \cdot 1$	$-35+1 = -34$

Ex 5) $x^2 + 4x - 12 = (x-2)(x+6)$

Product = -12	Sum = 4
$3 \cdot -4$	$2+-4 = -1$
$-3 \cdot 4$	$-3+4 = 1$
$-6 \cdot 2$	$-6+2 = -4$
$-2 \cdot 6$	$-2+6 = 4$

Ex 6) $x^2+2x+1 = (x+1)(x+1) = (x+1)^2$

Product = 1	Sum = 2
$1 \cdot 1$	$1+1=2$

Since x^2+2x+1 is equal to something squared then x^2+2x+1 is a perfect square.

Ex 7) $x^2-8x+16 = (x-4)(x-4) = (x-4)^2$

perfect square

Product	Sum
$4 \cdot 4$	$4+4=8$
$-4 \cdot -4$	$-4+-4=-8$
$8 \cdot 2$	$8+2=10$
$-8 \cdot -2$	$-8+-2=-10$

Ex 8) x^2+3x+5 = Prime

Product	Sum
$1 \cdot 5$	$1+5=6$
$-5 \cdot -1$	$-5+-1=-6$

Nothing works so this cannot be factored

Ex 9) $x^5+5x^3+x^2+5$ - from web assign

$x^5+x^2+5x^3+5$

$x^2(x^3+1)+5(x^3+1)$

$(x^2+5)(x^3+1)$ - but this is not a correct answer in web assign since (x^3+1) is still factorable.

Sum and Difference of cubes

Binomial — Trinomials

$(x^3+8) = (x+2)(x^2-2x+4)$

cube roots (x) (2)

opposites

Square the 1st cube root

cube roots multiplied

Square the 2nd cube root

$(x^3-8) = (x-2)(x^2+2x+4)$

cube roots (x) (2)

opposites

Remember our chart numbers

$1^3 = 1$ $4^3 = 64$

$2^3 = 8$ $5^3 = 125$

$3^3 = 27$ $6^3 = 216$

Back to web assign

$x^5 + 5x^3 + x^2 + 5 = (x^2+5)(x^3+1)$

$(x^2+5)(x+1)(x^2-x+1)$

S	O	A	P
Same	Opposite	Always	Positive

Use S.O.A.P to remember the signs for Sum and difference of cubes

Ex 10) Web assign

Factor Completely

$216t^3 - 8 = (6t-2)(36t^2+12t+4)$

cube roots (6t) (2)

They share common factors so we are not done yet

$= 2(3t-1)(4)(9t^2+3t+1)$

$= 8(3t-1)(9t^2+3t+1)$

Problems worked on the board

1) $x^2 - 2x + 24 = (x-6)(x+4)$

Product = 24	Sum = -2
$2 \cdot -12$	$2 + -12 = -10$
$-2 \cdot 12$	$-2 + 12 = 10$
$-3 \cdot 8$	$-3 + 8 = 5$
$3 \cdot -8$	$3 + -8 = -5$
$-6 \cdot 4$	$-6 + 4 = -2$
$6 \cdot -4$	$6 + -4 = 2$

2) $x^2 + 25x + 24 = (x+1)(x+24)$

Product = 24	Sum = 25
$1 \cdot 24$	$1 + 24 = 25$
$-1 \cdot -24$	$-1 + -24 = -25$
$3 \cdot 8$	$3 + 8 = 11$
$-3 \cdot -8$	$-3 + -8 = -11$

3) $x^2 + 11x + 24 = (x+3)(x+8)$

Product = 24	Sum = 11
$3 \cdot 8$	$3 + 8 = 11$
$-3 \cdot -8$	$-3 + -8 = -11$

4) $x^2 - 5x - 24 = (x-8)(x+3)$

Product = 24	Sum = -5
$-3 \cdot 8$	$-3 + 8 = 5$
$-8 \cdot 3$	$-8 + 3 = -5$