Step 1: Replace big numbers with chart numbers
If number is on the chart more than once, choose the exponent that matches the denominator of the fractional exponent

1) \[(\frac{16}{625})^{\frac{3}{4}}\] = \[(\frac{2^4}{5^4})^{\frac{3}{4}}\] = \[\frac{2^\frac{4 \cdot \frac{3}{4}}{5^\frac{4 \cdot -\frac{3}{4}}}}{5^{\frac{4 \cdot -\frac{3}{4}}}}\]
   = \[\frac{2^{-3}}{5^{-3}}\]
   = \[\frac{8^3}{2^3}\] = \[\frac{125}{8}\]

2) \[(-27)^{-\frac{1}{3}}\] = \[(-\frac{1}{3^3})^{-\frac{1}{3}}\] = \[\frac{-\frac{1}{3^3}}{-\frac{1}{3}}\] = \[-\frac{1}{3}\] = \[-3\] = \[\frac{-3}{1}\]

3) \[(\frac{1}{\sqrt[5]{32}})^{-\frac{2}{5}}\] = \[(\frac{1}{\sqrt[5]{2^5}})^{-\frac{2}{5}}\] = \[(\frac{1}{2})^{-\frac{2}{5}}\]
   = \[(2^{-\frac{5}{2}})^{-\frac{2}{5}}\]
   = \[2^{\frac{5}{2} \cdot -\frac{2}{5}}\]
   = \[2\]
   = \[2^1 = 2\]
Simplifying Radical Expressions

**Rule I.** No radicand may contain a factor to a power greater than or equal to the index of the radical.

Ex 1) \( \sqrt{75} \)

Is this simplified? Yes or No

Let's make a factor tree to find the prime factors of 75.

```
  75
   \_\_\_
     3   25
     \_\_\_\_
       5   5
```

So \( \sqrt{75} = \sqrt{3 \cdot 5 \cdot 5} \)

Since the index is 2 and 5 shows up 2 times then \( \sqrt{75} \) is not simplified.

\[ \sqrt{2 \cdot 3 \cdot 5 \cdot 5} = 5 \sqrt{3} \]

Our number is now simplified since no number under the square root appears equal too or greater than the index number of times.

Ex 2) \( \frac{2 \sqrt{27x^3}}{} \)

\[ \frac{2 \sqrt{3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x}}{} = 3x \frac{2 \sqrt{3}x}{\phantom{x}} \]
Ex 3)

\[
\sqrt[3]{5^4 a^3 b^4} = \sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b}
\]

Since the index is 3 then we need to take out things that appear 3 times.

\[
= 3a b \sqrt[3]{2b}
\]

Ex 4) Webassign: Question

Q1:

\[
\sqrt[8]{24 x y^8} = \sqrt[2]{2 \cdot 3 \cdot 2 \cdot 2 \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}
\]

\[
= 2 y y y y \sqrt[2]{2 \cdot 3 \cdot x}
\]

\[
= 2 y^4 \sqrt[2]{6 x}
\]

Q2:

\[
\sqrt[8]{108 a^8 b^2} = \sqrt[2]{2 \cdot 3 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b}
\]

It's a rule that \(\sqrt[2]{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}}\)

So then we can use this.

Factor tree

\[
\frac{108 a^8}{b^2} = \frac{108 a^2}{b^2} = \frac{2 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a}{b^2 \cdot \sqrt[3]{3}}
\]

\[
= \frac{2 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a}{b^2 \cdot \sqrt{3}}
\]
Q 2: continued

\[ \frac{6a^4 \sqrt{2}}{b} \]

This is the correct answer but WebAssign wants there to be absolute values.

WebAssign correct answer

\[ \frac{6a^4 \sqrt{2}}{|b|} \]

Why does WebAssign say \( \sqrt{b^2} = |b| \)?

Let's check

1) \( b \) is positive

\[ \sqrt{b^2} = b \quad b = 5 \]

\[ \sqrt{25} = 5 \quad \checkmark \]

2) \( b \) is negative

\[ \sqrt{b^2} = b \quad b = -5 \]

\[ \sqrt{25} = 5 \quad \checkmark \]

\[ \text{but } | \sqrt{b^2} | = 5 \]

So, \( \sqrt{b^2} = |b| \) because \( b \) can be positive or negative, since \(( -5 )^2 = 5^2 \)

\[ -5 \cdot -5 = 5 \cdot 5 \]

\[ 25 = 25 \]
Rule II. No power of the radicand and the index of the radical may have a common factor other than one.

Ex 1: \[ \sqrt[6]{a^4} \] Since they share a common factor of 2 we divide the index and the power by 2.

\[ \sqrt[6]{a^4} = \sqrt[3]{\sqrt[2]{a^2}} = \sqrt[3]{a^2} \]

Ex 2: \[ \sqrt[9]{a^3 \cdot b^6} = \sqrt[3]{a^{3/3} \cdot b^{6/3}} = \sqrt[3]{a \cdot b^2} \]

Ex 3: \[ \sqrt[4]{a^3 \cdot b^6 \cdot c^7} \] Even though 9, 3, and 6 all have a common factor we cannot do anything because all 4 numbers need to share a common factor.