IV. Rational (Fractional) Exponents

\[ 8^{\frac{3}{2}} = \left(\frac{8}{\sqrt{2}}\right)^2 \]

\[ 27^{\frac{4}{3}} = \left(\sqrt[3]{27}\right)^4 = \left(\sqrt[3]{3^4}\right) \]

\[ \sqrt[3]{8} = 2 \quad \text{since} \quad 2 \cdot 2 \cdot 2 = 8 \]

\[ 8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4 \]

Calculator: \[ 8 \times \left(\frac{8}{\sqrt{2}}\right)^2 = 4 \]

\[ 64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2 = 64^{\frac{2}{3}} = 16 \]

\[ a^{b/c} = \left(\sqrt[c]{a^b}\right)^c \quad \text{or} \quad \left(\sqrt[c]{a}\right)^{bc} \]

\[ a^{-b} = \frac{1}{a^b} \]

\[ a^{-\frac{1}{2}} \]

\[ \sqrt{7} = 7^{\frac{1}{2}} \]

\[ x^0 = 1 \quad \text{if there's no visible exponent, it is 1st power.} \]

\[ \sqrt[n]{a} \] index is the same as "square root" 2.
Never have a negative exponent as the final answer.

RATIONAL (FRACTIONAL) EXPONENTS are challenging.
- Exponents are fractions, e.g. $8^{2/3}$
- Translate rational exponents into radical expression.
- Definitions:
  1. Radicand: the stuff under the radical
  2. Index:
  3. A radical expression is asking you what number may be used as the index number of times results in the radicand.

   Formula:
   $$a^{b/c} = \left(\sqrt[c]{a}\right)^b = \left(\sqrt[a]{a^b}\right)$$
   $$\sqrt[2]{x^b} = x^{b/2}$$

   Example: $\sqrt[3]{7} = 7^{1/3} \neq \sqrt[7]{7}$
   $\sqrt[2]{7} = \sqrt{7}$

   If there is no visible exponent, then the exponent is understood as 1.
   If there is no visible index on the radical expression, it's understood as 2.