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Factoring Polynomials

I. Always factor out the largest common factor first

II. Consider factoring by grouping (use this when you see a polynomial with 4 terms)

III. Consider the difference of two squares

remember in the last section we saw conjugates multiplied together

Ex: $(x - 5)(x + 5) = x^2 - 5x + 5x - 25 = x^2 - 25$

So now looking at the difference of two squares

$$x^2 - 25 = (x + 5)(x - 5)$$

one binomial will always have a + and one will have a -.

we will always have 2 binomials multiplied together for the factors of a difference of two squares.

Ex: $16x^2 - 9 = (\quad + \quad)(\quad - \quad)$ ← we know that its 2 binomials 1 with + and 1 with -

$= (4x + 3)(4x - 3)$ ← we then put the factors of $16x^2$ in front and 9 in the back.

Ex: $4xy^2 - 4xz^2 = 4x(y^2 - z^2)$ - always factor out the biggest common factor out first

$$= 4x(y+z)(y-z)$$

Ex: $16x^2 - 25z^2 = (4x + 5z)(4x - 5z)$

Ex: $x^4 - 25 = (x^2 - 5)(x^2 + 5)$

Ex: $a^{10} - 64 = (a^5 - 8)(a^5 + 8)$

Ex: $y^{26} - 4 = (y^{13} - 2)(y^{13} + 2)$

Notice that all of the perfect squares have even exponents, like y^{26} , a^{10} , x^4 , ...

Ex: $x^{20}y^{30} - 36 = (x^{10}y^{15} - 6)(x^{10}y^{15} + 6)$

Ex: $x^2 + 25 =$

↑

Is it $(x+5)(x+5) = x^2 + 5x + 5x + 25$? X

Is it $(x-5)(x-5) = x^2 - 5x - 5x + 25$? X

Since we cannot factor this polynomial then it is prime.

It's a + not a -
so it's not a
difference of
squares.

Sum of squares are prime

Ex: $(x^2 + 25)$, $(z^4 + 16)$, $(y^8 + 36)$, etc.

Factor completely

Ex: $x^4 - 16 = (x^2 + 4)(x^2 - 4) \leftarrow$
↓ ↓
the same since $(x^2 + 4)$ is prime
 $= (x^2 + 4)(x - 2)(x + 2)$

Notice that $(x^2 - 4)$ is a difference of squares so it factors

Examples from webassign

Ex: $-8x^3 - 56x = 8x(x^2 - 7)$

— factor out largest common factor. Notice 7 is not a square so we are done

Ex: $18 - 50z^2 = 2(9 - 25z^2)$
↓
 $= 2(3 - 5z)(3 + 5z)$

— $(9 - 25z^2)$ is the difference of squares so we can factor it.

Notice 18 and 50 are not squares so we need to factor out the greatest common factor

Ex: $(x - 3)^2 - 25 = ((x - 3) - 5)((x - 3) + 5)$ } simplify

Remember $a^2 - b^2 = (a - b)(a + b) = (x - 8)(x + 2)$

if it looks the same
if $a = (x - 3)$ and $b = 5$