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**Factoring Polynomials**

I. Always factor out the largest common factor first.

II. Consider factoring by grouping (use this when you see a polynomial with 4 terms).

III. Consider the difference of two squares.

Remember in the last section we saw conjugates multiplied together:

\[(x - 5)(x + 5) = x^2 - 5x + 5x - 25 = x^2 - 25\]

So, now looking at the difference of two squares:

\[x^2 - 25 = (x + 5)(x - 5)\]

One binomial will always have a + and one will have a -.

Ex: \[16x^2 - 9 = ( + )( - )\]

\[= (4x + 3)(4x - 3)\]

We know that its 2 binomials 1 will + and 1 will -.

We then put the factors of 16x^2 in front and -9 in the back.
Ex: \[ 4x^2y^2 - 4xz^2 = 4x(y^2 - z^2) \] Always factor out the biggest common factor out first.

\[ = 4x(y+z)(y-z) \]

Ex: \[ 16x^2 - 25z^2 = (4x + 5z)(4x - 5z) \]

Ex: \[ x^4 - 25 = (x^2 - 5)(x^2 + 5) \]

Ex: \[ a^{10} - 64 = (a^5 - 8)(a^5 + 8) \]

Ex: \[ y^{26} - 4 = (y^{13} - 2)(y^{13} + 2) \]

Notice that all of the perfect squares have even exponents, like \( y^2 \), \( a^{10} \) and \( x^4 \).

Ex: \[ x^{20}y^{30} - 36 = (x^{10}y^{15} - 6)(x^{10}y^{15} + 6) \]

Ex: \[ x^2 + 25 \]

\[ \uparrow \]

Is it \( (x+5)(x-5) = x^2 + 5x - 5x + 25 ? \]

Is it \( (x-5)(x+5) = x^2 - 5x + 5x - 25 ? \]

Since we cannot factor this polynomial into factors, then it is prime.
Sum of squares are prime

Ex: \((x^2 + 28), (z^4 + 16), (y^8 + 36), \text{ etc.}\)

Factor completely

Ex: \(x^4 - 16 = (x^2 + 4)(x^2 - 4) \downarrow \text{ Notice that } (x^2 - 4) \text{ is a difference of squares so it factors.} \downarrow \)

\(\uparrow \text{ stays the same since } (x^2 + 4) \text{ is prime.} \downarrow \)

Examples from Webassign

Ex: \(8x^3 - 56x = 8x(x^2 - 7) \text{ - factor out largest common factor. Notice 7 is not a square so we are done.} \)

Ex: \(18 - 50z^2 = 2(9 - 25z^2) \text{ - } (9 - 25z^2) \text{ is the difference of squares so we can factor it.} \)

Notice \(18 \text{ and } 50z^2 \text{ are not squares, so we need to factor out the greatest common factor.} \)

Ex: \((x - 3)^2 - 25 = ((x - 3) - 5)(x - 3) + 5) \text{ - simplify.} \)

Remember \(a^2 - b^2 = (a - b)(a + b) = (x - 8)(x + 2) \text{ if it looks the same.} \)

if \(a = (x - 3) \text{ and } b = 5\).