

(1) $\sqrt{18x^3} = \sqrt{2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x} = 3x\sqrt{2x}$

Diagram: A tree diagram showing 18 as 2 * 9, and 9 as 3 * 3. The 3s and x's are circled in the original image.

something under the radical will appear TWICE (2)
bring out the 3x IN FRONT OF RADICAL AS A FACTOR,
not as index ($\sqrt[3]{2x}$ is wrong)

(2) $\sqrt[3]{40a^8b^4} = \sqrt[3]{5 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b} = 2ab\sqrt[3]{5b}$

Diagram: A tree diagram showing 40 as 5 * 8, and 8 as 2 * 2 * 2.

(3) $\sqrt[9]{a^3b^6} = \sqrt[9/3]{a^{3/3}b^{6/3}} = \sqrt[3]{ab^2}$

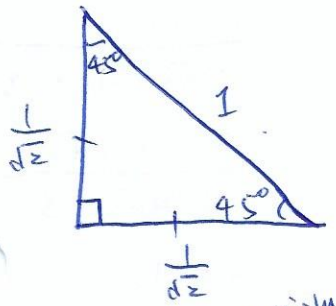
Index and inside the radicals share a common factor

Two Types of Problems
(1) something under the radical appear "index" # of times
(2) something under the radical share a common factor with "index"

(4) $\sqrt[6]{27x^3} = \sqrt[6]{3^3x^3} = \sqrt[6/3]{3^{3/3}x^{3/3}} = \sqrt[2]{3x} = \sqrt{3x}$

a chart number.

III. NO RADICAL MAY APPEAR IN THE DENOMINATOR



- Right triangle.
- isosceles → two sides of equal length.
- If the hypotenuse = 1, then each sides on the right angle has length $\frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Diagram: A box around $\frac{\sqrt{2}}{\sqrt{2}}$ with an arrow pointing to it from the text "multiply by 1".

To get rid of the radical in the denominator, multiply this number by the "radical/radical" = 1, to change how it looks.

• Rationalization of Denominator,

- change the denominator from irrational to rational,
- the whole thing still remains ^{as an} irrational number.

$$\frac{\cancel{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{\boxed{3}}$$

↑ irrational
 ↑ rational denominator

~~square~~
square root

* you multiple a " $\sqrt{\quad}$ " by itself, you get just the radicant (aka what's under the radical)

◦ - here in this example, we just rewrite the irrational number, $\frac{2}{\sqrt{6}}$, as $\frac{\sqrt{6}}{3}$. They are the same number but we've rationalized the denominator from irrational $\sqrt{6}$ to rational 3.

e.g. $\frac{2}{2+\sqrt{3}}$

Multiply the number by $\frac{2-\sqrt{3}}{2-\sqrt{3}}$

$$\begin{aligned} & \frac{2}{2+\sqrt{3}} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})} \rightarrow \text{"CONJUGATES"} \\ & = \frac{2 \cdot 2 - 2 \cdot \sqrt{3}}{2 \cdot 2 - 2\sqrt{3} + 2\sqrt{3} - 3} \rightarrow \text{add to zero.} \\ & = \frac{4 - 2\sqrt{3}}{4 - 3} \\ & = \frac{4 - 2\sqrt{3}}{1} = \boxed{4 - 2\sqrt{3}} \end{aligned}$$

These are wrong simplification strategies

(1) $\frac{2}{2+\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\boxed{2\sqrt{3}} + 3}$ ← still has a radical on bottom
distributive property

(2) $\frac{2}{2+\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2(2+\sqrt{3})}{2+2\sqrt{3}+2\sqrt{3}+\sqrt{3} \cdot \sqrt{3}}$
f.o.i.l.
 $= \frac{2 \cdot 2 + 2 \cdot \sqrt{3}}{2 + 4\sqrt{3} + 3}$

$= \frac{4 + 2\sqrt{3}}{5 + \boxed{4\sqrt{3}}}$
still has a radical here

eg. $\frac{5}{\sqrt{7} + \sqrt{2}}$

$= \frac{5}{\sqrt{7} + \sqrt{2}} \cdot \frac{\sqrt{7} - \sqrt{2}}{\sqrt{7} - \sqrt{2}}$ = 1

→ conjugate

$= \frac{5 \cdot (\sqrt{7} - \sqrt{2})}{\sqrt{7} \cdot \sqrt{7} - \sqrt{2} \cdot \sqrt{7} + \sqrt{2} \cdot \sqrt{7} - \sqrt{2} \cdot \sqrt{2}}$

$= \frac{5(\sqrt{7} - \sqrt{2})}{7 - \cancel{\sqrt{14}} + \cancel{\sqrt{14}} - 2}$

$= \frac{5(\sqrt{7} - \sqrt{2})}{7 - 2}$

$= \frac{\cancel{5}(\sqrt{7} - \sqrt{2})}{\cancel{5}}$

$= \boxed{\sqrt{7} - \sqrt{2}}$

STRATEGY.

- recognize the expression for the denominator involves the addition/subtraction of at least one radical.
- multiply this radical expression by its conjugate, meaning, change the sign between the two numbers to get the conjugate
- the middle term ~~is~~ of $(a+b)(a-b)$ will always add up to zero.

