1/28 Exponents & Radical Expressions

I. Positive Integer Exponents Ex: 2^3

II. Zero as an Exponent Ex: 2^0

III. Negative integer Exponents Ex: 2^{-3}

IV. Rational (Fractional) Exponents Ex: $8^{2/3}$, $27^{-4/3}$

I. Positive Integer Exponents

Ex: $2^{\overset{\text{exponent or power}}{3}}$ → base

$2^3 \rightarrow 2 \cdot 2 \cdot 2 = 8$

$3x^5$ ~~$(3x)(3x)(3x)(3x)(3x)$~~ Not correct

$3 \cdot x \cdot x \cdot x \cdot x \cdot x$ Correct

$3X^5$ → exponent; X → base; 3 → coefficient

The guiding principal for how to distinguish what the base is that the base is the factor immediately next to the exponent unless ()'s tell you otherwise.

Ex: $5xy^3 = 5 \cdot x \cdot y \cdot y \cdot y$

$5(xy)^3 = 5(xy)(xy)(xy)$

$(-2)^4 = (-2)(-2)(-2)(-2)$ (power: 4; base: -2)

$-2^4 = -1(2 \cdot 2 \cdot 2 \cdot 2)$ (coefficient of -2: -1; base: 2; base multiplied 4 times)

A negative number to an even power will be positive

A negative number to an odd power will be negative

$(Neg)^{even} = Pos$

$(Neg)^{odd} = Neg$

Example Quiz question

$(-1)^{431} =$

A. 431
B. -431
C. 1
D. -1

Correct Answer: D. -1

First Property of Exponents

$x^3 \cdot x^2 = (x \cdot x \cdot x) \cdot (x \cdot x) = (x \cdot x \cdot x \cdot x \cdot x) = x^5$

$x^3 \cdot x^2 \rightarrow x^{3+2}$

Same base

$a^7 \cdot a^3 \cdot a = a^{7+3+1} = a^{11}$ since $a = a^1$

$3ab^2 \cdot 5a^2b^2 \cdot 6ab^3 = (3 \cdot 5 \cdot 6)(a \cdot a^2 \cdot a)(b^2 \cdot b^2 \cdot b^3) = 90(a^{1+2+1})(b^{2+2+3})$

$= 90a^4b^7$

Second Property of Exponents

$(x^3)^2 = x^{3 \cdot 2} = x^6$

$(x^3)^2$ is known as a power of a power

$(x^3)^2 = (x^3)(x^3) = x^6$

$(xy^2z^3)^5 = x^5y^{10}z^{15}$

$(2a^3)^3 = 2^{1 \cdot 3}a^{3 \cdot 3} = 8a^9$

$2^3 \cdot 2^4 =$

A. 4^{12}

B. 4^7

C. 2^{12}

D. 2^7

Since we are multiplying we add the powers so the power will be 7. Also since the base will not change we are multiplying seven 2's so the answer is D.

II. Zero as an Exponent

$2^0 = 1$

$2^3 \cdot 2^0 = 2^{3+0} = 2^3 = 8$

$8 \cdot \underline{1} = 8$

So the blank must be 1 since no other number will work

$7^0 = 1$ $\quad 13.5^0 = 1$ $\quad \pi^0 = 1$ $\quad \left(\frac{1}{19}\right)^0 = 1$ $\quad (-37.2)^0 = 1$

Any real number to the zero power is 1.

$-7^0 = -1(7^0) = -1(1) = -1$

$(-7)^0 = 1$

Example Problems from WebAssign

$$\left(-\frac{3}{4}\right)^3\left(\frac{4}{3}\right)^2 = \left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)\left(\frac{4}{3}\right)\left(\frac{4}{3}\right) = \frac{-3}{4}$$

negative since neg to odd power.

you can cancel