DERIVATIVES

If $(c, f(c))$ is the point of tangency and $(c+\Delta x, f(c+\Delta x))$ is a second point on the graph of f, then the <u>slope of the secant line</u> through the two points is

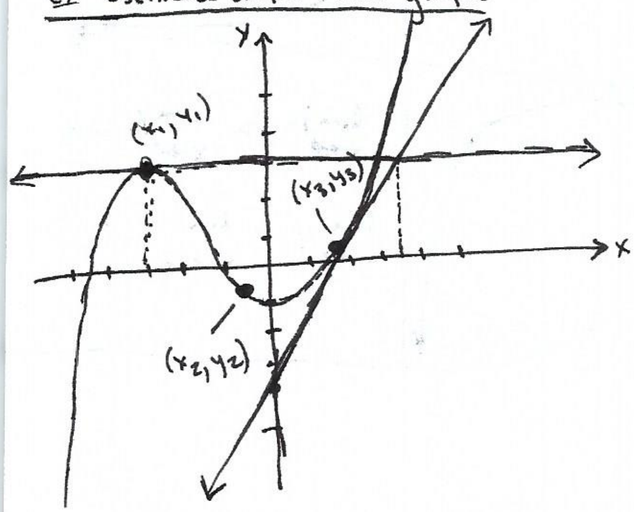
$$m = \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

If f is defined on an open interval containing c, and if

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = m,$$

then the line passing through $(c, f(c))$ with slope m is tangent to f at $(c, f(c))$

<u>Ex</u> Estimate slope from graph



$@ (x_1, y_1)$: $\frac{3-3}{-3-0} = 0$

$@ (x_3, y_3)$: $\frac{3-(-4)}{3.5-0} = \frac{7}{3.5} = 2$

<u>Ex.</u> Use definition to find slope of tangent line to $f(x) = x^2 - 1$ at $(0, -1)$ and $\left(\begin{array}{l} f(x) = x^2+1 \\ \text{at } (-1, 2) \end{array} \right)$

$(0,-1)$: $m = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{c+\Delta x - c} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(\Delta x)^2 - 1 - (0^2 - 1)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{(\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} (\Delta x)$

$(-1, 2)$: $m = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} \underset{\lim_{\Delta x \to 0}}{=} \frac{(-1+\Delta x)^2 + 1 - (\overbrace{(-1)^2+1}^{2})}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[1^2 + (\Delta x)^2 - 2\Delta x - 1\right]$

$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[(\Delta x)^2 - 2\Delta x\right] = \lim_{\Delta x \to 0} \left[\Delta x - 2\right] = -2$

If f continuous at c and $\lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = \pm\infty$, then there is a <u>vertical tangent line</u> at $(c, f(c))$

The <u>derivative</u> of f is given by:

@ x: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ or

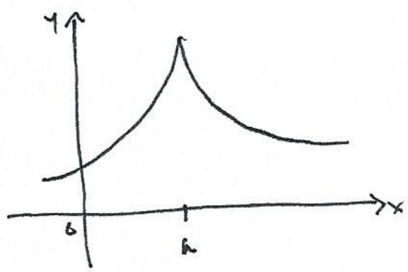
@ $x=c$: $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

* This is also known as the <u>instantaneous rate of change</u>.

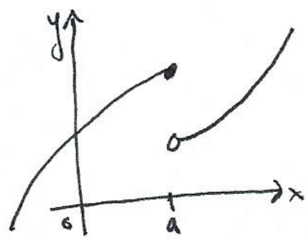
10 Sep 2018

Notations for derivative: y' $f'(x)$ $\frac{dy}{dx}$ $\frac{d}{dx}[f(x)]$ $D_x y$

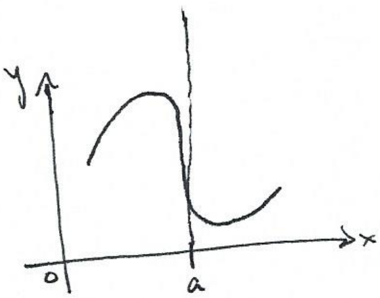
Three ways for f to not be differentiable:



corner



break



vertical tangent

Ex Compute $g'(t)$ for $g(t) = \frac{2}{t}$

$$g'(t) = \lim_{\Delta t \to 0} \frac{g(t+\Delta t) - g(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\frac{2}{t+\Delta t} - \frac{2}{t}}{\Delta t} = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left[\frac{2t - 2t - 2\Delta t}{t(t+\Delta t)}\right] = \lim_{\Delta t \to 0} \frac{-2\Delta t}{t(t+\Delta t)} \frac{1}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{-2}{t(t+\Delta t)} = \frac{-2}{t^2}$$