Ex. $g(x) = \dfrac{x}{x^2+1}$ x continuous
x^2+1 continuous and never equal to 0 for $x \in \mathbb{R}$

Then $g(x)$ continuous on $\mathbb{R}$.

Ex. Find constants a, b so that $f(x)$ continuous on $\mathbb{R}$,

with $f(x) = \begin{cases} 2, & x \le -1 \\ ax+b, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$

* need to check continuity @ $x = -1, 3$

So we need $\lim\limits_{x \to -1} f(x) = f(-1)$

and $\lim\limits_{x \to 3} f(x) = f(3)$

$\lim\limits_{x \to -1} f(x) = f(-1) = 2$

$\lim\limits_{x \to -1^-} 2 = \lim\limits_{x \to -1^+} (ax+b) = -a+b = 2$

$\longrightarrow b = a+2 = (-1)+2 = +1$

$\lim\limits_{x \to 3} f(x) = f(3) = -2$

$\lim\limits_{x \to 3^+} (-2) = \lim\limits_{x \to 3^-} (ax+b) = 3a+b = -2$

$= 3a + (a+2) = 4a+2 = -2$

$4a = -4$

$a = -1$

So $f(x) = \begin{cases} 2, & x \le -1 \\ 1-x, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$

5 Sep 2018

Intermediate Value Theorem

If f is continuous on $[a,b]$, $f(a) \ne f(b)$, and $f(a) < k < f(b)$, then there is a number c in $[a,b]$ such that $f(c) = k$.

Ex. Show that $f(x) = x^3 + 2x - 1$ has a zero on $[0,1]$.

$f(0) = 0^3 + 2\cdot 0 + (-1) = -1$
$f(1) = 1^3 + 2\cdot 1 - 1 = 2$

$\longrightarrow f(a) \ne f(b)$

$f(x)$ is a polynomial
$\longrightarrow f$ continuous

$0 \in (-1, 2)$, so by IVT there exists $c \in [0,1]$ such that $f(c) = 0$

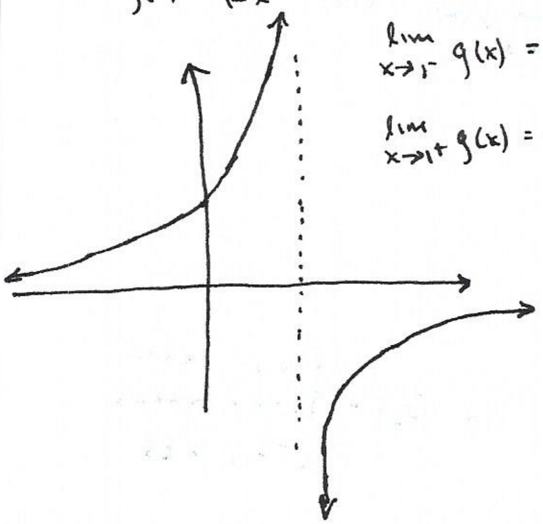
Infinite Limits

Math 2413
Dr. Liu
5 Sep 2018
pg 12

An infinite limit is a limit in which $f(x)$ increases or decreases without bound as x approaches c; $\lim_{x \to c} f(x) = \infty$.

$\lim_{x \to c} f(x) = \infty$ does not mean that the limit exists; it tells you how the limit fails to exist by denoting the unbounded behavior of $f(x)$ as x approaches c.

Ex $g(x) = \frac{1}{1-x}$



$\lim_{x \to 1^-} g(x) = \infty$

$\lim_{x \to 1^+} g(x) = -\infty$

The line $x = c$ is a vertical asymptote if at least one of the following is true:

$\lim_{x \to c} f(x) = \pm\infty \qquad \lim_{x \to c^+} f(x) = \pm\infty \qquad \lim_{x \to c^-} f(x) = \pm\infty$

Ex. Find any vertical asymptotes.

1) $g(x) = \cot(x) = \frac{\cot(x)}{\sin(x)}$

$\sin(x) = 0$

$x = k\pi,\ k \in \mathbb{Z}$

vertical asymptotes here

2) $l(x) = xe^{-2x} = \frac{x}{e^{2x}}$

$e^{2x} = 0$

$2x = \ln(0) = \text{DNE}$

no vertical asymptotes

3) $s(x) = \ln(x^2 - 4)$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

Math 2413
Dr Lin
5 Sep 2018
pg 13

4) $h(x) = \dfrac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$

$= \dfrac{(x-5)(x+3)}{(x-5)(x^2+1)}$

$= \dfrac{x+3}{x^2+1}$

$x^3 - 5x^2 + x - 5 = 0$

$x^2(x-5) + (x-5) = (x-5)(x^2+1) = 0$

$x - 5 = 0$ $\qquad$ $x^2 + 1 = 0$

$\underline{x = 5}$ $\qquad$ $x^2 = -1$

no real solution

Since we can factorize the numerator to cancel out the zero in the denominator at $x=5$, this point is a removable discontinuity, <u>not</u> a vertical asymptote.

<u>Thm</u> Let c and L be real numbers and let f and g be functions such that

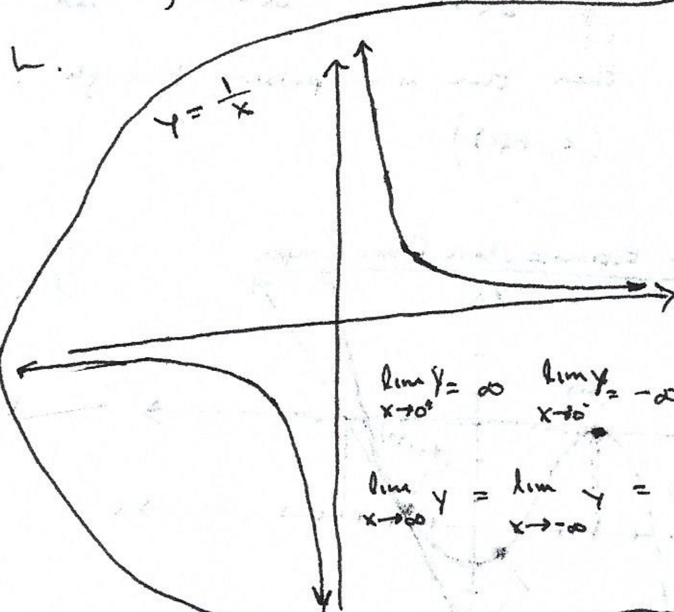
$$\lim_{x \to c} f(x) = \infty \quad \text{and} \quad \lim_{x \to c} g(x) = L.$$

Then:

1) $\displaystyle \lim_{x \to c} \left[f(x) \pm g(x) \right] = \infty$

2) $\displaystyle \lim_{x \to c} \left[f(x) g(x) \right] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3) $\displaystyle \lim_{x \to c} \frac{g(x)}{f(x)} = 0$



$\displaystyle \lim_{x \to 0^+} y = \infty \qquad \lim_{x \to 0^-} y = -\infty$

$\displaystyle \lim_{x \to \infty} y = \lim_{x \to -\infty} y = 0$

<u>Ex.</u> $\displaystyle \lim_{x \to 1^-} \frac{x^2 - 3x}{x - 1} \longrightarrow \frac{\text{constant neg \#}}{\text{something goes to 0}} \longrightarrow -\infty$

$(+1)^2 - 3(+1) = 1 - 3 = -2$

<u>Ex.</u> $\displaystyle \lim_{x \to 0} \left[1 + \frac{1}{x^2} \right] = \infty$

<u>Ex.</u> $\displaystyle \lim_{x \to -3^-} \frac{x}{\sqrt{x^2 - 9}} = \frac{-\#}{\text{something goes to 0}} = -\infty$

<u>Ex.</u> $\displaystyle \lim_{x \to -3^-} \frac{x+3}{x^2 + x - 6} = \lim_{x \to -3^-} \frac{x+3}{(x-2)(x+3)} = \lim_{x \to -3^-} \frac{1}{x-2} = \frac{-1}{5}$

<u>Recall:</u> $\displaystyle \lim_{x \to c} f(x) = \infty$ really means that the limit does not exist because $f(x)$ increases or decreases without bound

<u>Ex.</u> $\displaystyle \lim_{x \to 0^-} \left(x^2 + \frac{1}{x} \right) = 0 - \infty = \infty$

<u>Ex.</u> $\displaystyle \lim_{x \to \frac{\pi}{2}^-} \ln \left| \cos(x) \right| = \text{DNE}$

$\cos\left(\frac{\pi}{2}\right) = 0$

<u>Ex.</u> $\displaystyle \lim_{x \to 0^+} \left(6 - \frac{1}{x^3} \right) = 6 - \infty = -\infty$