4. $g(x) = \arctan(x^2)$ on $[-2,1]$

$g'(x) = \dfrac{2x}{1+(x^2)^2} = \dfrac{2x}{1+x^4}$

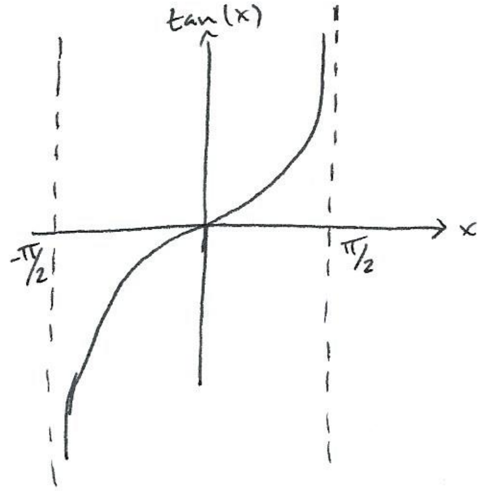
$g'(x) = 0 \implies \dfrac{2x}{1+x^4} = 0$

$x = 0$

$g'(x)$ DNE $\implies 1 + x^4 = 0$

$x^4 = -1$

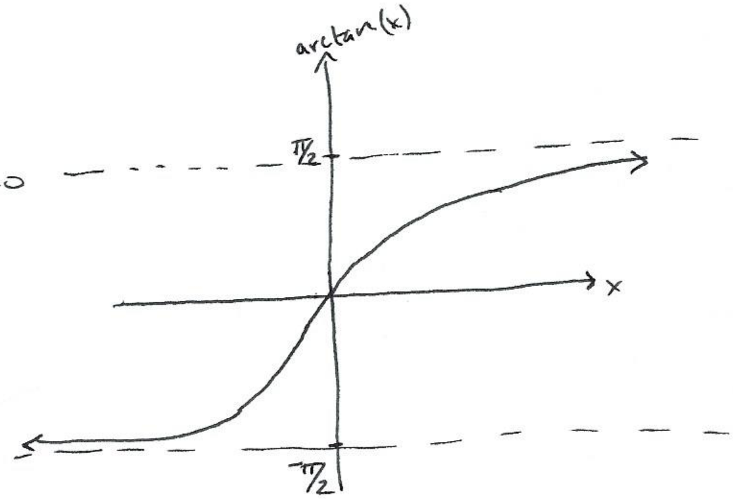
no such x



$g(0) = \arctan(0) = 0$, so g defined @ $x=0$

$g(-2) = \arctan(4)$

$g(1) = \arctan(1)$



$g(0) = 0 \longleftarrow$ abs min

$g(-2) = \arctan(4) \longleftarrow$ abs max

5. $h(x) = x^3 - 3x^2 + 1$ on $[-1,1]$

$h'(x) = 3x^2 - 6x$

$h'(x) = 0 \implies 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0$ or $x = 2$

But 2 is not in $[-1,1]$.

$h'(x)$ DNE $\implies$ no solution since h' is a polynomial and continuous on $[-1,1]$

Since h is a polynomial, h is defined at $x=0$.

$h(0) = 1 \longleftarrow$ abs max

$h(-1) = -1 - 3 + 1 = -3 \longleftarrow$ abs min

$h(1) = +1 - 3 + 1 = -1$

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#6 $\ell(x) = 5e^{x} - e^{2x}$ on $[-1,2]$

$\ell'(x) = 5e^{x} - 2e^{2x}$

$\ell'(x) = 0 \Rightarrow 5e^{x} - 2e^{2x} = 0$, $e^{x} > 0$ for all x

$5 - 2e^{x} = 0$

$e^{x} = \frac{5}{2}$

$x = \ln\left(\frac{5}{2}\right) \approx 0.916$

$\ell'(x)$ DNE $\Rightarrow$ no solution because exponential function defined for all x

$\ell(-1) = \frac{5}{e} - \frac{1}{e^2}$

$\ell(+2) = 5e^{2} - e^{4}$

$\ell(\ln(\frac{5}{2})) = 5e^{\ln(\frac{5}{2})} - e^{2\ln(\frac{5}{2})} = 5 \cdot \frac{5}{2} - \left(\frac{5}{2}\right)^2 = \frac{25}{2} - \frac{25}{4} = \frac{25}{4} = 6.25$