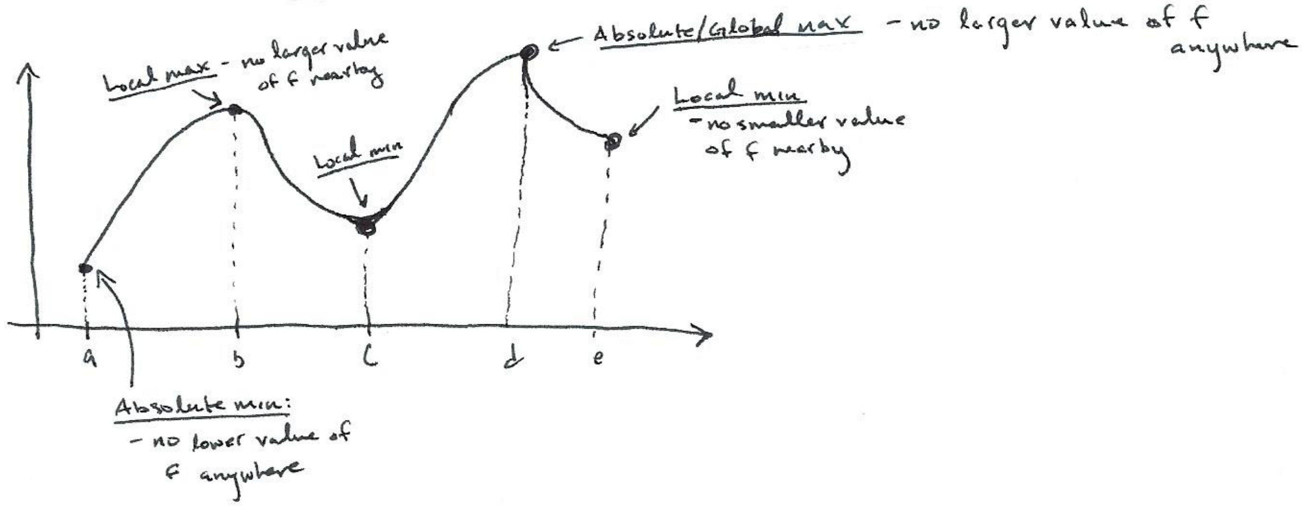
Extrema

Def. Let f be defined on an interval containing c.

1) $f(c)$ is the minimum of f on I when $f(c) \le f(x)$ for all x in I.

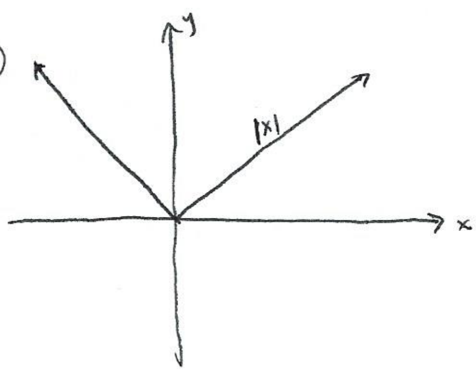
2) " " maximum " " $f(c) \ge f(x)$ " "

Thm. If f is continuous on the closed interval $[a,b]$, then f has both a minimum and a maximum on the interval.



Ex. Find the value of the derivative at each extremum

(1)

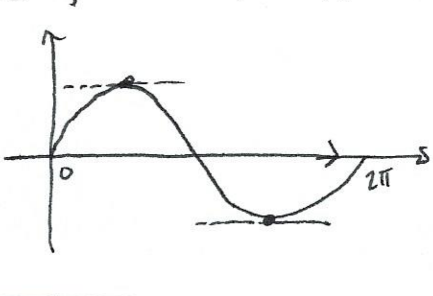


$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{-x-0}{x-0} = \lim_{x\to 0} \frac{-x}{x} = \lim_{x\to 0^-} (-1) = -1$$

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{x-0}{x-0} = \lim_{x\to 0^+} (1) = 1$$

* An extremum can occur where the derivative does not exist

(2) $g(x) = \sin(x)$, $(\frac{\pi}{2}, 1)$, $(\frac{3\pi}{2}, -1)$



$g'(x) = \cos(x)$

$\cos(\frac{\pi}{2}) = \cos(\frac{3\pi}{2}) = 0$

* An extremum can occur where the derivative is equal to 0

Def. Let f be defined at c. If $f'(c) = 0$ or if $f'(c)$ does not exist, then c is a critical point of f.

Ex. Find the critical points of $f(t) = 2t\ln(t)$

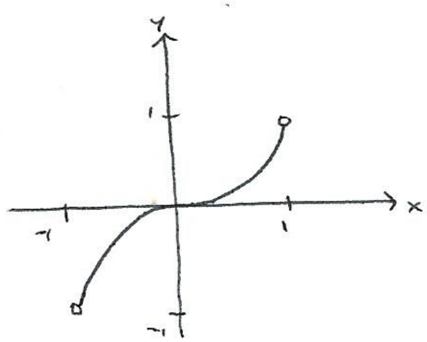
$$f'(t) = 2\ln(t) + 2t\,\frac{1}{t} = 2(\ln(t) + 1)$$

$$f'(t) = 0 \Rightarrow 2\ln(t) = -2$$
$$\ln(t) = -1$$
$$t = e^{-1} = \frac{1}{e} \quad \leftarrow f(t) \text{ is defined at } \frac{1}{e}$$

$f'(t)$ DNE for $t \le 0$, but $f(t)$ undefined for $t \le 0$.

Then the only critical point is $t = \frac{1}{e}$.

Ex. Approximate critical numbers and determine whether the functions has
- absolute min/max
- relative min/max



Since the function not defined at the endpoints, there is no absolute min/max.

The first derivative $= 0$ at $x = 0$, but f has no local min/max here.

Note. To find extreme of continuous function f on closed interval $[a, b]$:

1) Find critical numbers of f in (a, b)
2) Evaluate f at each critical number
3) Evaluate f at each endpoint of $[a, b]$.
4) The smallest of these is the minimum
   largest maximum

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Dr. Liu
27 Sep 2018
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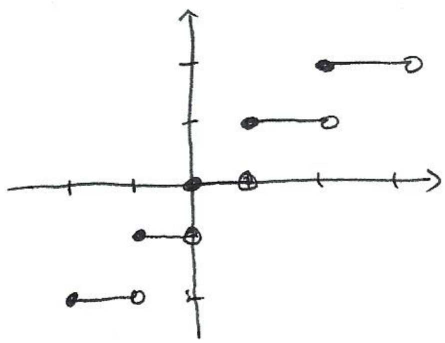
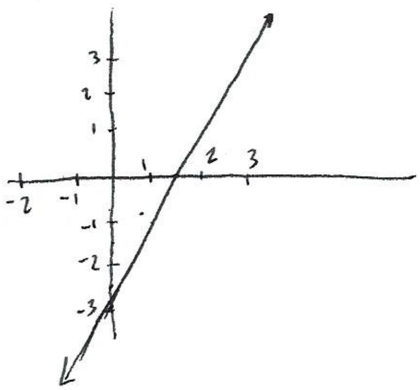
Ex Find the extrema

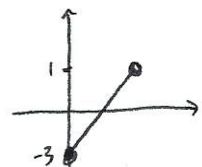
1) $m(x) = [\![x]\!]$ on $[-2,2]$



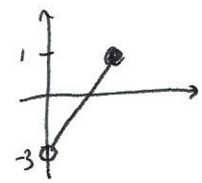
rel/abs min : -2
rel/abs max : 2

2) $n(x) = 2x-3$ on $[0,2)$, $(0,2]$, $(0,2)$

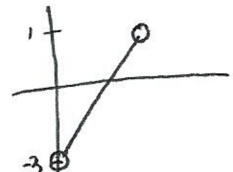




$[0,2)$



$(0,2]$



$[0,2)$	$(0,2]$	$(0,2)$
local/abs max DNE	local/abs max $x=2$, $n(x)=1$	Local/abs max DNE
local/abs min $x=0$ $n(x)=-3$	local/abs min DNE	local/abs min DNE

3) $f(x) = 2x - 3x^{2/3}$ on $[-1,3]$

$f'(x) = 2 - \frac{2}{3}3x^{-1/3}$

$= 2 - 2x^{-1/3}$

$f'(x) = 0 \implies 2 = 2x^{-1/3}$

$1 = x^{-1/3}$

$1 = x$

$f'(x)$ DNE $\implies x = 0$

Since f defined at $x=0$ and $x=1$, these are both critical numbers

$f(0) = 0 \leftarrow$ abs max

$f(1) = -1$

$f(-1) = -2 - 3(-1)^{2/3} = -2 - 3 = -5 \leftarrow$ abs min

$f(3) = 6 - 3(3^2)^{1/3} = 6 - 3 \cdot 9^{1/3}$