Derivatives of Inverse Functions

Let $f$ be a function whose domain is an interval $I$. If $f$ has an inverse function $f^{-1}$, then:

1) If $f$ is continuous on its domain, then $f^{-1}$ continuous on its domain.

2) If $f$ is differentiable on an interval containing $c$ and $f'(c) \neq 0$, then $f^{-1}$ is differentiable at $f(c)$.

Moreover,
\[
(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}
\]

Ex.
Let $f(x) = \frac{1}{4}x^3 + x - 1$.

1) \( f^{-1}(3) = ? \)

2) \( (f^{-1})'(3) = ? \)

1) \( f^{-1}(3) = a \iff 3 = f(a) \)

\[
\frac{1}{4}x^3 + x - 1 = 3 \\
x^3 + 4x = 16 \\
x = 2 = f^{-1}(3)
\]

2) \( (f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(2)} = \frac{1}{4} \)

\( f'(x) = \frac{3}{4}x^2 + 1 \)

\( f'(2) = \frac{3}{4}(4) + 1 = 3 + 1 = 4 \)