Derivatives of Inverse Functions

Math 2413
Dr. Lu
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Thm: Let f be a function whose domain is an interval I. If f has an inverse function f^{-1}, then:

1) If f is continuous on its domain, then f^{-1} continuous on its domain
2) If f is differentiable on an interval containing c and $f'(c) \neq 0$, then f^{-1} is differentiable at $f(c)$.

Thm: Let f be a function differentiable on an interval I. If f has an inverse function f^{-1}, then f^{-1} is differentiable at any x for which $f'(f^{-1}(x)) \neq 0$.

Moreover, $$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Ex. Let $f(x) = \frac{1}{4}x^3 + x - 1$.

1) $f^{-1}(3) = ?$
2) $(f^{-1})'(3) = ?$

1) $f^{-1}(3) = a \iff 3 = f(a)$

$$\frac{1}{4}x^3 + x - 1 = 3$$
$$x^3 + 4x = 16$$
$$x = 2 = f^{-1}(3)$$

2) $$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(2)} = \frac{1}{4}$$

$$f'(x) = \frac{3}{4}x^2 + 1$$
$$f'(2) = \frac{3}{4}(4) + 1 = 3 + 1 = 4$$