

Ex. Find $\frac{d^2y}{dx^2}$

$$x^2 + y^2 = 25$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = -\frac{1 \cdot y - x \cdot \frac{dy}{dx}}{y^2} = -\frac{y - x \left(-\frac{x}{y} \right)}{y^2} = -\frac{y^2 + x^2}{y^3} = -\frac{25}{y^3}$$

Ex. Find $\frac{dy}{dx}$ implicitly for $\sin(y) = x$. Then find the largest interval $-a < y < a$ on which y is a differentiable function of x .

$$\sin(y) = x$$

$$\cos^2(y) + \sin^2(y) = 1$$

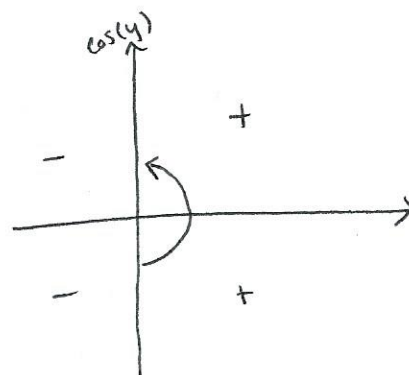
$$\cos(y) = \sqrt{1 - \sin^2(y)} = \sqrt{1 - x^2}$$

$$\cos(y) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)} = (1 - x^2)^{-1/2}$$

$$-a < y < a$$

$$-\frac{\pi}{2} < y < \frac{\pi}{2}$$



Ex. Find the tangent line of $x^2(x^2 + y^2) = y^2$ at the point $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

$$x^2(x^2 + y^2) = y^2$$

$$x^4 + x^2y^2 = y^2$$

$$x^4 + (x^2 - 1)y^2 = 0$$

$$4x^3 + 2xy^2 + 2(x^2 - 1)y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x^3 + 2xy^2}{2(x^2 - 1)y}$$

$$\left. \frac{dy}{dx} \right|_{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)} = \frac{4 \left(\frac{\sqrt{2}}{2} \right)^3 + 2 \left(\frac{\sqrt{2}}{2} \right)^3}{2 \left(1 - \left(\frac{\sqrt{2}}{2} \right)^2 \right) \frac{\sqrt{2}}{2}} = \frac{\left(\frac{\sqrt{2}}{2} \right)^2 (4 + 2)}{2 - 2 \left(\frac{\sqrt{2}}{2} \right)^2} = \frac{\left(\frac{2}{2} \right) (6)}{2 - 2 \left(\frac{2}{4} \right)} = 3$$

$$\underline{y - \frac{\sqrt{2}}{2} = 3 \left(x - \frac{\sqrt{2}}{2} \right)}$$