5) $m(x) = \begin{cases} x^2+1, & x \le 2 \\ 4x-3, & x > 2 \end{cases}$

$$\lim_{x\to 2^-} \frac{m(x)-m(2)}{x-2} = \lim_{x\to 2^-} \frac{x^2+1-2^2-1}{x-2} = \lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2^-} (x+2) = 4$$

$$\lim_{x\to 2^+} \frac{m(x)-m(2)}{x-2} = \lim_{x\to 2^+} \frac{4x-3-8+3}{x-2} = \lim_{x\to 2^+} \frac{4x-8}{x-2} = \lim_{x\to 2^+} 4\cdot\frac{(x-2)}{(x-2)} = 4$$

Then m is differentiable at $x=2$.

Basic Differentiation Rules

Let c be a real number and let n be a rational number. If f and g are differentiable functions, then:

a) $(c)' = 0$

b) $(x^n)' = nx^{n-1}$

c) $(cf(x))' = cf'(x)$

d) $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

Also, $(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(e^x)' = e^x$

Ex. Find the derivative of the function

1) $g(x) = \sqrt[4]{x^5} - \frac{6}{x^3} + x^{-2} + 4$

$= x^{4/5} - 6x^{-3} + x^{-2} + 4$

$g'(x) = \frac{4}{5}x^{-1/5} + 18x^{-4} - 2x^{-3}$

2) $h(x) = \frac{3}{2\sqrt[3]{x^2}} - \frac{1}{5}\sin x + 3e^x - \pi^2$

$= \frac{3}{2}x^{-2/3} - \frac{1}{5}\sin x + 3e^x - \pi^2$

$h'(x) = -x^{-5/3} - \frac{1}{5}\cos x + 3e^x$

3) $l(x) = \frac{3x^2 - x + 1}{x} - \frac{1}{2}\cos x$

$= 3x - 1 + \frac{1}{x} - \frac{1}{2}\cos x$

$= 3x - 1 + x^{-1} - \frac{1}{2}\cos x$

$= 3 - x^{-2} + \frac{1}{2}\sin x$

Ex. Find an equation of the tangent line to the graph of $g(t) = \sin t + \frac{1}{2}e^t$ at $t = \pi$.

$g'(t) = +\cos t + \frac{1}{2}e^t$

$g'(\pi) = +\cos\pi + \frac{1}{2}e^\pi = -1 + \frac{1}{2}e^\pi$

$g(\pi) = \sin\pi + \frac{1}{2}e^\pi = \frac{1}{2}e^\pi$

$\mapsto$ @ $(\pi, \frac{1}{2}e^\pi)$, $y - \frac{1}{2}e^\pi = (-1 + \frac{1}{2}e^\pi)(x - \pi)$

is the equation of the line tangent to g

Ex. Determine the points (if any) at which the graph of the function $f(x) = \frac{1}{x^2} = x^{-2}$ has a horizontal tangent line

$f(x) = x^{-2}$

$f'(x) = -2x^{-3}$

$-2x^{-3} = 0$

$x^{-3} = 0$

$\frac{1}{x^3} = 0 \Rightarrow$ no such x $\Rightarrow$ $f(x) = \frac{1}{x^2}$ does not have a horizontal tangent line

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Ex Find k such that the line $y = x + 4$ is tangent to the graph of the function

$f(x) = k\sqrt{x}$

$f'(x) = (kx^{1/2})' = \frac{k}{2}x^{-1/2}$

At $x = c$, slope of tangent line to $f(x) = k\sqrt{x}$

is $\frac{kc^{-1/2}}{2} = 1$.

$k = 2c^{1/2}$

$k = 2(4)^{1/2}$

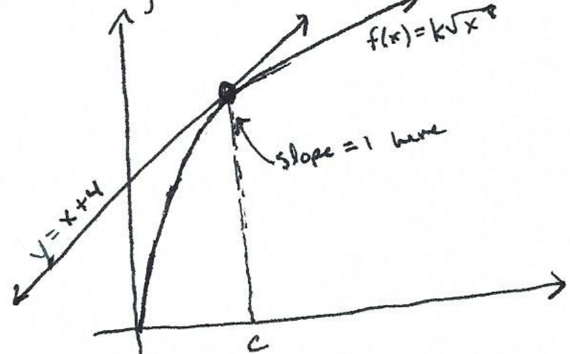
$\underline{k = 4}$

$c + 4 = k\sqrt{c}$

$= 2c^{1/2}c^{1/2} = 2c$

$c + 4 = 2c$

$\underline{4 = c}$



* In this example, we had <u>two</u> unknowns, so we needed <u>two</u> equations to solve for those unknowns *

The function s that gives the position of an object with respect to the origin as a function of time is called the <u>position function</u>. If, over a period of time $\Delta t = t_2 - t_1$ the object changes position by amount $\Delta s = s(t + \Delta t) - s(t)$ or $\Delta s = s(t_2) - s(t_1)$,

then, since rate $= \frac{\text{distance}}{\text{time}}$, the <u>average velocity</u> is $\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$.

If the object is moving in a straight line, then the velocity or <u>instantaneous velocity</u> is

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = s'(t)$$