Math 2413
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pg 16

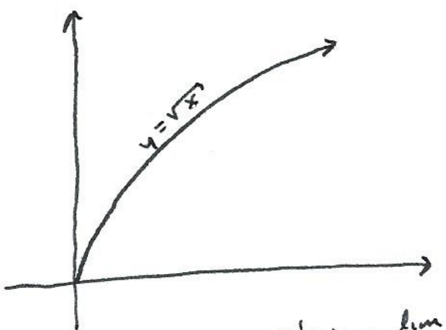
Ex. Find the equation of the tangent line of $f(x) = \sqrt{x}$ at the point $(4,2)$.
Discuss the behavior of f at $(0,0)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[f(x+\Delta x) - f(x)\right] = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[\sqrt{x+\Delta x} - \sqrt{x}\right]$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[\left(\sqrt{x+\Delta x} - \sqrt{x}\right)\frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}\right] = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[\frac{x+\Delta x - x}{\sqrt{x+\Delta x} + \sqrt{x}}\right]$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}} \longmapsto f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

$= \text{slope of tangent line}$



$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{4}(x-4) = \frac{1}{4}x - 1$

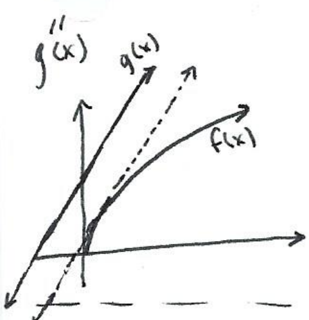
$y = \frac{1}{4}x + 1$ is the equation of the line tangent to $f(x) = \sqrt{x}$ at $(4,2)$

at $x=0$, $f'(x) = \lim_{x \to 0} \frac{1}{2\sqrt{x}} = \infty \longmapsto f$ is not differentiable at $x=0$

Ex. Find an equation of the line tangent to $f(x) = \sqrt{x}$ and parallel to the line $2x - y + 1 = 0$

$2x - y + 1 = 0$

$y = 2x + 1 \to \text{slope} = 2 = \text{slope of needed tangent line} = f'(x) = \frac{1}{2\sqrt{x}}$ for some $x > 0$.



$2 = \frac{1}{2\sqrt{x}}$

$4\sqrt{x} = 1$

$\sqrt{x} = \frac{1}{4} = f\left(\frac{1}{16}\right)$

$x = \frac{1}{16}$

$y - \frac{1}{4} = 2\left(x - \frac{1}{16}\right) = 2x - \frac{1}{8}$

$y = 2x - \frac{1}{8} + \frac{2}{8} = 2x + \frac{1}{8}$

$y = 2x + \frac{1}{8}$ is tangent to f at $\left(\frac{1}{16}, \frac{1}{4}\right)$ and is parallel to $g(x)$

Ex. Find the equation of the line tangent to $f(x) = \sqrt{x}$ and perpendicular to the line $2x + y + 1 = 0$

$2x + y + 1 = 0$

$g(x) = y = -2x - 1$

$\to$ slope $= -2$

$\to$ slope of tan line $= \frac{1}{2} \longmapsto \frac{1}{2} = f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2} = \frac{1}{2\sqrt{x}}$

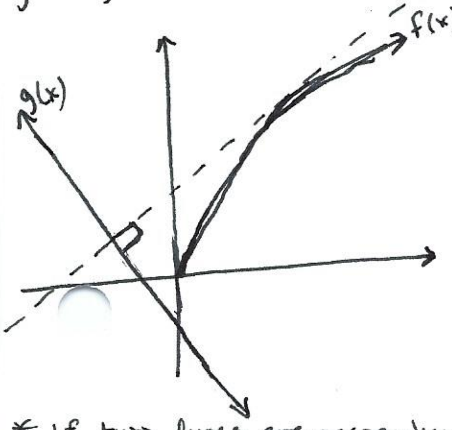
$\sqrt{x} = 1 = f(1)$

$x = 1$

$y - 1 = \frac{1}{2}(x-1)$

$y = \frac{1}{2}x + \frac{1}{2} = \frac{1}{2}(x+1)$

$y = \frac{1}{2}(x+1)$ is tangent to $f(x)$ at $(1,1)$ and perpendicular to $g(x)$.



* If two lines are perpendicular, the product of their slopes is (-1). *

Math 2413
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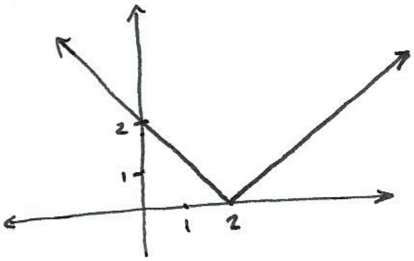
11 Sep 2018
pg 17

Thm If f is differentiable at $x=c$, then f is continuous at $x=c$.
If f not continuous at $x=c$, then f is not differentiable at $x=c$.

E Determine whether the function is differentiable at the given point.

1) $f(x) = |x-2|$ at $x=2$



f not diff at $x=2$ b/c there is a corner, which indicates that $\lim\limits_{x\to 2^+} f \neq \lim\limits_{x\to 2^-} f$, which means $\lim\limits_{x\to 2} f$ does not exist.

show numerically: $\lim\limits_{x\to 2^-} \dfrac{f(x)-f(2)}{x-2} = \dfrac{-(x-2)-0}{x-2} = -1$

$\lim\limits_{x\to 2^+} \dfrac{f(x)-f(2)}{x-2} = \dfrac{(x-2)-0}{x-2} = 1$

Since $\lim\limits_{x\to 2^-} f = -1 \neq 1 = \lim\limits_{x\to 2^+} f$, f not continuous at $x=2$, so f not differentiable at $x=2$.

$$f'(2) = \lim_{\Delta x\to 0} \frac{f(2+\Delta x)-f(2)}{\Delta x} = \lim_{\Delta x\to 0} \frac{1}{\Delta x}\left(|2+\Delta x-2| - |2-2|\right)$$

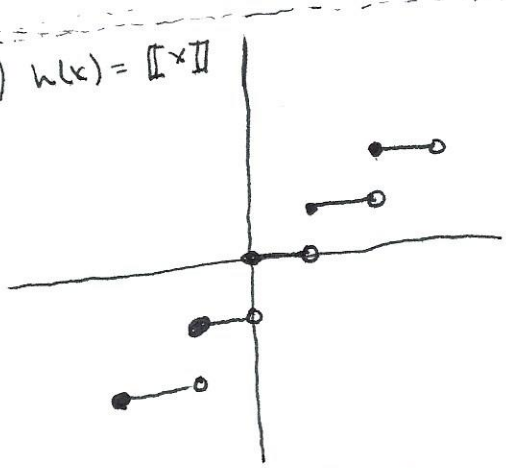
$$= \lim_{\Delta x\to 0} \frac{|\Delta x|}{\Delta x} = \begin{cases} 1, & \Delta x>0 \\ -1, & \Delta x<0 \end{cases}$$

2) $g(x) = x^{1/3}$ at $x=0$

$\lim\limits_{x\to 0} \dfrac{g(x)-g(0)}{x-0} = \lim\limits_{x\to 0} \dfrac{x^{1/3}}{x} = \lim\limits_{x\to 0} \dfrac{1}{x^{2/3}} = \lim\limits_{x\to 0} \dfrac{1}{(x^2)^{1/3}} = \infty$, so the limit does not exist.

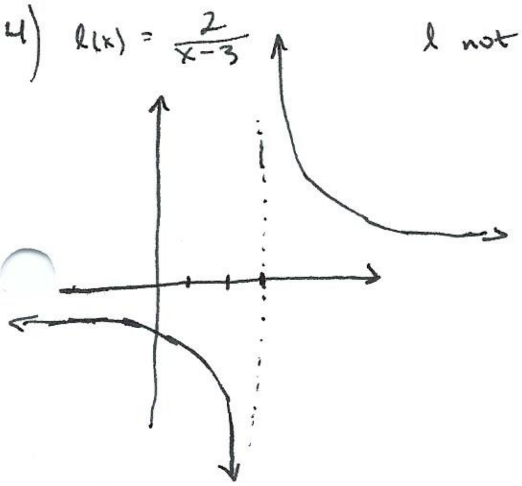
Since the limit does not exist, the function is not differentiable at the point.

3) $h(x) = [\![x]\!]$



Since h not continuous at $x=0$, h not differentiable at $x=0$.

4) $l(x) = \dfrac{2}{x-3}$



l not continuous at $x=3$, so l not differentiable at $x=3$.