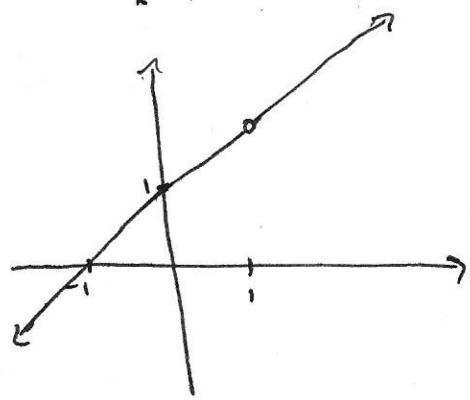
Note 2: If the function factors and the bottom term cancels, the discontinuity at the x-value for which the denominator was zero was removable, so the graph has a hole in it.
If not, the discontinuity is non removable and the graph has a vertical asymptote.

Ex. $f(x) = \dfrac{x^2-1}{x-1} = \dfrac{(x+1)(x-1)}{x-1} = x+1 \longrightarrow$ removable discontinuity @ $x=1$



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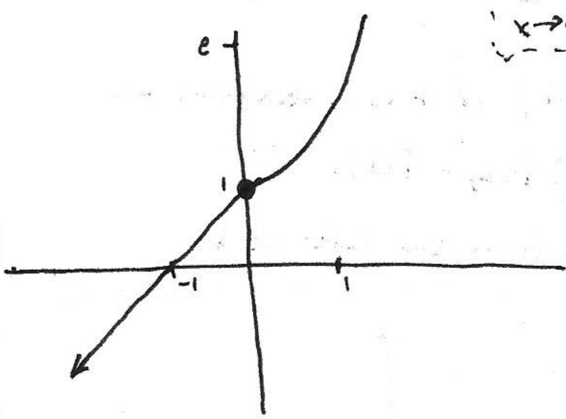
Ex. $h(x) = \begin{cases} x+1, & x \le 0 \\ e^x, & x > 0 \end{cases}$

$\lim\limits_{x\to 0^-} (x+1) = 1$
$\lim\limits_{x\to 0^+} e^x = 1$

$\longrightarrow$ the limits agree, so $h(0)=1$ and we have a solid dot at $(0,1)$.

$\longrightarrow$ so $\lim\limits_{x\to 0} h(x) = h(0) = 1$,
so $h(x)$ is continuous at $x=0$.



when $x \le 0$, $h(x) = x+1$, which is everywhere continuous. when we approach 0, we come from the left and we write $\lim\limits_{x\to 0^-} h(x)$.
the minus sign means "from the left"

when $x > 0$, $h(x) = e^x$, which is everywhere continuous. we approach 0 from the right, and we write

$\lim\limits_{x\to 0^+} h(x)$.
the plus sign means "from the right."

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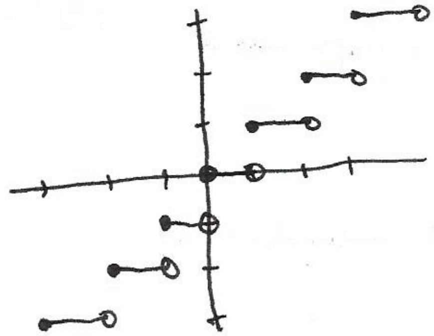
Ex. $\lim_{x\to 0^-} [\![x]\!]$ and $\lim_{x\to 0^+} [\![x]\!]$



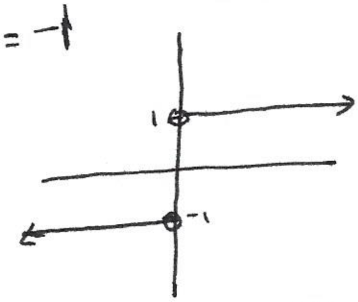
$\lim_{x\to 0^-} [\![x]\!] = -1$

$\lim_{x\to 0^+} [\![x]\!] = 0$

Then $\lim_{x\to 0} [\![x]\!]$ does not exist.

Ex. $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-}(-1) = -1$

$$\frac{|x|}{x} = \begin{cases} -1, & x<0 \\ DNE, & x=0 \\ 1, & x>0 \end{cases}$$



Ex $\lim_{x\to 4^+} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4^+} \frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x\to 4^+} \frac{x-4}{x-4}\frac{1}{\sqrt{x}+2} = \lim_{x\to 4^+}\frac{1}{\sqrt{x}+2} = \frac{1}{2+2} = \frac{1}{4}$

We can say that $\lim_{x\to c} f(x) = L$ if and only if $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$

The function f is continuous on the closed interval $[a,b]$ if f is continuous on the open interval (a,b) and $\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$.

Then f is continuous from the right at and continuous from the left at b.

Ex. Discuss continuity of $f(x) = \sqrt{1-x^2}$ on $[-1,1]$

$f(-1) = 0$ $\quad$ $f(1) = 0$

$\lim_{x\to -1^+} f(x) = 0$ $\quad$ $\lim_{x\to 1^-} f(x) = 0$

So f is continuous on $[-1,1]$.