Math 2413
Dr. Lim
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Ex $\lim_{x\to 1} \frac{x^2-x}{x^2+3x-4} = \lim_{x\to 1} \frac{(x-1)x}{(x-1)(x+4)} = \lim_{x\to 1} \frac{x}{x+4} = \frac{1}{5}$

Ex $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} = \lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3} = \lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$

$= \lim_{x\to 4} \frac{x-4}{x-4} \cdot \frac{1}{\sqrt{x+5}+3} = \lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

Ex $\lim_{x\to 0} \frac{\frac{1}{3+x}-\frac{1}{3}}{x} = \lim_{x\to 0} \frac{\frac{3-(3+x)}{3(3+x)}}{x} = \lim_{x\to 0} \frac{1}{x} \cdot \frac{-x}{3(3+x)} = \lim_{x\to 0} \frac{-1}{9+3x} = \frac{-1}{9}$

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Squeeze Theorem: if $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if $\lim_{x\to c} h(x) = L = \lim_{x\to c} g(x)$, then $\lim_{x\to c} f(x)$ exists and is equal to L.

Theorem: $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ $\quad \lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$ $\quad \lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

Ex: Use Squeeze Theorem to find $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$

x^2 bounded below by 0. $\quad \sin\left(\frac{1}{x}\right)$ bounded below by -1 and above by 1

$0 \le x^2 < \infty$

$-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$

$\lim_{x\to 0}(-x^2) = 0 \qquad \lim_{x\to 0} x^2 = 0$

Then $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$ exists and is equal to 0.

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Ex Find $\lim\limits_{x\to 0} \frac{\tan(x)}{x} = 1$

$$\lim_{x\to 0} \frac{\tan(x)}{x} = \lim_{x\to 0} \frac{\sin(x)}{x}\cdot\frac{1}{\cos(x)} = \left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{1}{\cos x}\right) = 1\cdot 1 = 1$$

Ex. Find $\lim\limits_{x\to 0} \frac{\sin(4x)}{x} = 4$

$$\lim_{x\to 0}\frac{\sin(4x)}{x} = \lim_{x\to 0} 4\cdot\frac{\sin(4x)}{4x} = 4\cdot\lim_{x\to 0}\frac{\sin(4x)}{4x} = 4(1) = 4$$

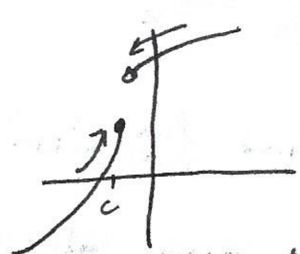
Ex. $$\lim_{x\to 0}\frac{1-e^{-x}}{e^{x}-1} = \lim_{x\to 0}\frac{e^{x}}{e^{x}}\frac{1-e^{-x}}{e^{x}-1} = \lim_{x\to 0}\frac{1}{e^{x}}\frac{e^{x}-1}{e^{x}-1} = \lim_{x\to 0}\frac{1}{e^{x}} = 1$$

Ex. $$\lim_{x\to 0}\frac{e^{2x}-1}{e^{x}-1} = \lim_{x\to 0}\frac{(e^{x})^2-1}{e^{x}-1} = \lim_{x\to 0}\frac{(e^{x}+1)(e^{x}-1)}{e^{x}-1} = \lim_{x\to 0}(e^{x}+1) = 2$$

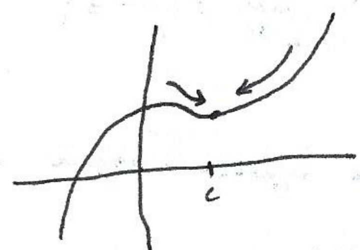
Continuity

A function f is continuous at c if and only if the following 3 conditions are satisfied:

1) $f(c)$ is defined
2) $\lim\limits_{x\to c} f(x)$ exists
3) $\lim\limits_{x\to c} f(x) = f(c)$



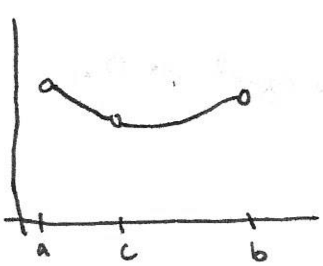
not continuous at c

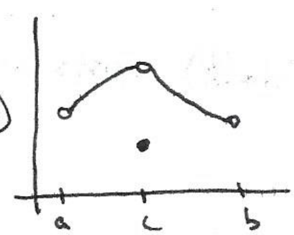


✓ continuous at c

* we say there is a discontinuity at c

Discontinuities can be either removable or nonremovable.

①


②


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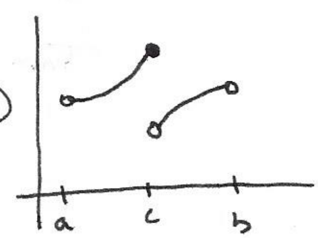
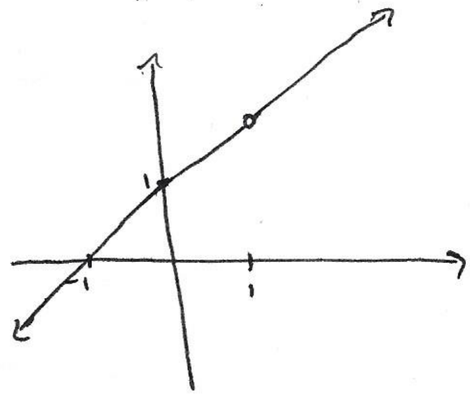


① and ② have removable discontinuities since we can define or redefine $f(c)$ to satisfy the continuity criteria.

③ The left and right limits do not agree, so this discontinuity is nonremovable.

<u>Note</u>[2] If the function factors and the bottom term cancels, the discontinuity at the x-value for which the denominator was zero was removable, so the graph has a hole in it.
If not, the discontinuity is non removable and the graph has a vertical asymptote.

<u>Ex</u>. $f(x) = \dfrac{x^2-1}{x-1} = \dfrac{(x+1)(x-1)}{x-1} = x+1 \longrightarrow$ removable discontinuity @ $x=1$



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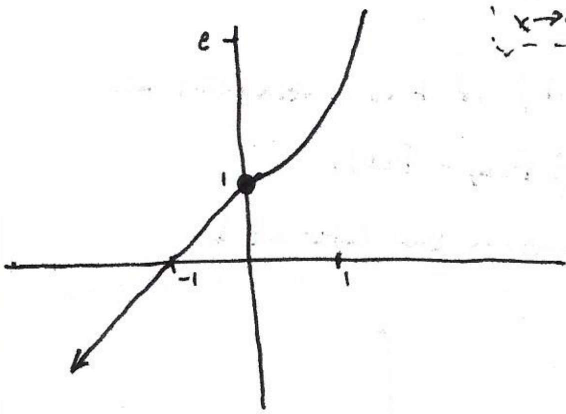
<u>Ex</u>. $h(x) = \begin{cases} x+1, & x \le 0 \\ e^x, & x > 0 \end{cases}$

$\lim_{x\to 0^-} (x+1) = 1$

$\lim_{x\to 0^+} e^x = 1$

$\longrightarrow$ the limits agree, so $h(0) = 1$ and we have a solid dot at $(0,1)$.

$\longrightarrow$ so $\lim_{x\to 0} h(x) = h(0) = 1$,
so $h(x)$ is continuous at $x=0$.



<u>when $x \le 0$,</u> $h(x) = x+1$, which is everywhere continuous.
when we approach 0, we come from the left and we write $\lim_{x\to 0^-} h(x)$.
the minus sign means "from the left"

<u>when $x > 0$</u>, $h(x) = e^x$, which is everywhere continuous.
we approach 0 from the right, and we write

$\lim_{x\to 0^+} h(x)$.
the plus sign means "from the right."