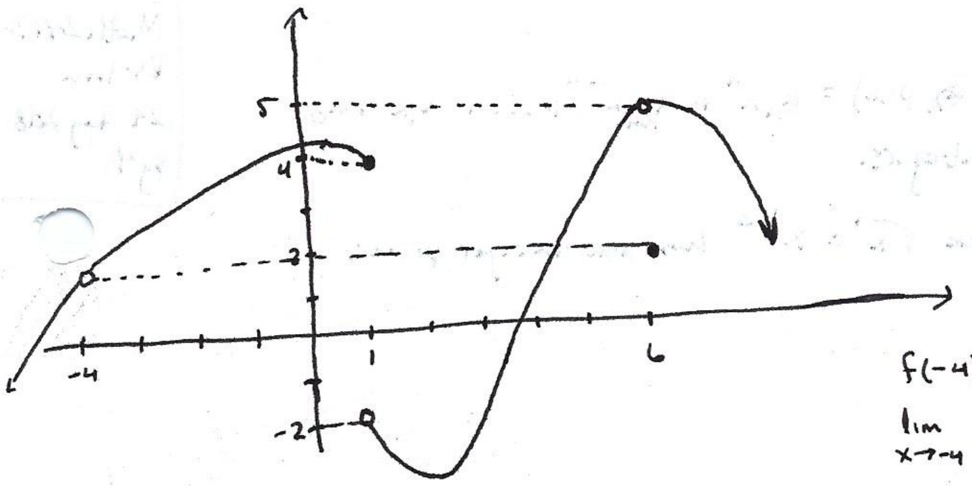

$f(-4) = \text{DNE}$

$\lim\limits_{x \to -4} f(x) = 2$

$f(1) = 4$

$\lim\limits_{x \to 1} f(x) = \text{DNE}$

$f(6) = 2$

$\lim\limits_{x \to 6} f(x) = 5$

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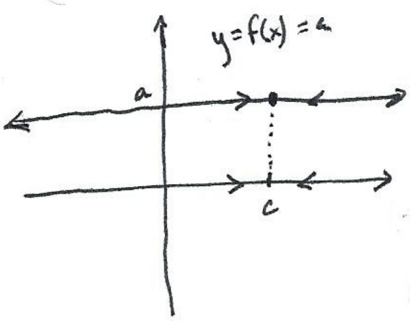
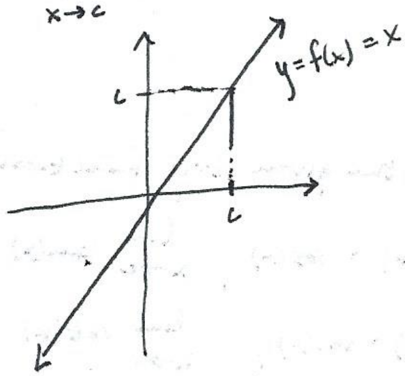
Evaluating Limits Analytically

- Some basic limits:

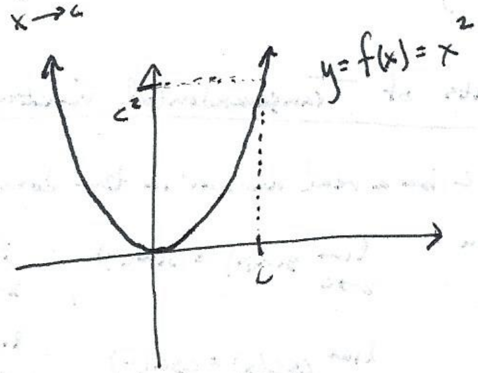
$\lim\limits_{x \to c} a = a$



$\lim\limits_{x \to c} x = c$



$\lim\limits_{x \to c} x^n = c^n$



Properties of Limits:

1) $\lim\limits_{x \to c} a \cdot f(x) = a \cdot \lim\limits_{x \to c} f(x) = a \cdot L$ ------ scalar multiple

2) $\lim\limits_{x \to c} \left[f(x) \pm g(x) \right] = \lim\limits_{x \to c} f(x) \pm \lim\limits_{x \to c} g(x) = L \pm K$ -- sum or difference

3) $\lim\limits_{x \to c} \left[f(x) \cdot g(x) \right] = \left[\lim\limits_{x \to c} f(x) \right] \cdot \left[\lim\limits_{x \to c} g(x) \right] = L \cdot K$ ---- product

4) $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to c} f(x)}{\lim\limits_{x \to c} g(x)} = \dfrac{L}{K},\ K \neq 0$ ------ quotient

5) $\lim\limits_{x \to c} \left(f(x) \right)^n = \left[\lim\limits_{x \to c} f(x) \right]^n = L^n$ ------ power

A polynomial $p(x)$ is written as $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$; the powers of x are always integers.

$f(x) = \sqrt{x}$ is not a polynomial since $\sqrt{x} = x^{1/2}$ has non-integer power.

* $\lim_{x \to c} p(x) = p(c)$

* $\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right)$

Ex $\lim_{x \to c} \left[5 \cdot f(x) \cdot g(x)\right]^2$, with $\lim_{x \to c} f(x) = 3$ and $\lim_{x \to c} g(x) = 2$

$$\lim_{x \to c}\left[5 \cdot f(x) \cdot g(x)\right]^2 = \left[\lim_{x \to c}\left(5 \cdot f(x) \cdot g(x)\right)\right]^2 = \left[\left(5 \cdot \lim_{x \to c} f(x)\right) \cdot \left(\lim_{x \to c} g(x)\right)\right]^2 = \left[5 \cdot (3) \cdot (2)\right]^2 = 900$$

Ex $f(x) = 5 - x$
$g(x) = x^3$

$$\lim_{x \to 1} g(f(x)) = \lim_{x \to 1} g(5-x) = \lim_{x \to 1} (5-x)^3 = \left(\lim_{x \to 1} (5-x)\right)^3 = (4)^3 = 64$$

Limits of Transcendental Functions

Let c be a real number in the domain of the given trig function.

Then
$\lim_{x \to c} \sin(x) = \sin(c)$, $\lim_{x \to c} \cos(x) = \cos(c)$, $\lim_{x \to c} \tan(x) = \tan(c)$

$\lim_{x \to c} \csc(x) = \csc(c)$, $\lim_{x \to c} \sec(x) = \sec(c)$, $\lim_{x \to c} \cot(x) = \cot(c)$

$\lim_{x \to c} \ln(x) = \ln(c)$ $\quad$ $\lim_{x \to c} a^x = a^c$, $a > 0$

Ex $\lim_{x \to e} \ln(x^3) = \ln(e^3) = 3\ln(e) = 3$

Ex $\lim_{x \to \frac{\pi}{6}} \cos(x)\sin^2(x) = \cos\left(\frac{\pi}{6}\right)\sin^2\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{\sqrt{3}}{8}$

recall: $\ln(x \cdot y) = \ln(x) + \ln(y)$
$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$, $y \neq 0$
$\ln(x^a) = a\ln(x)$
$y = \ln(x) \iff e^y = x$

Ex $\lim_{x\to 1} \frac{x^2-x}{x^2+3x-4} = \lim_{x\to 1} \frac{(x-1)x}{(x-1)(x+4)} = \lim_{x\to 1} \frac{x}{x+4} = \frac{1}{5}$

Ex $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} = \lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3} = \lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$

$= \lim_{x\to 4} \frac{x-4}{x-4} \cdot \frac{1}{\sqrt{x+5}+3} = \lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

Ex $\lim_{x\to 0} \frac{\frac{1}{3+x}-\frac{1}{3}}{x} = \lim_{x\to 0} \frac{\frac{3-(3+x)}{3(3+x)}}{x} = \lim_{x\to 0} \frac{1}{x} \cdot \frac{-x}{3(3+x)} = \lim_{x\to 0} \frac{-1}{9+3x} = \frac{-1}{9}$

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Squeeze Theorem: if $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if $\lim_{x\to c} h(x) = L = \lim_{x\to c} g(x)$, then $\lim_{x\to c} f(x)$ exists and is equal to L.

Theorem: $\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x\to 0} \frac{1-\cos(x)}{x} = 0 \qquad \lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

Ex: Use Squeeze Theorem to find $\lim_{x\to 0} x^2 \sin(\frac{1}{x})$

x^2 bounded below by 0. $\quad \sin(\frac{1}{x})$ bounded below by -1 and above by 1

$0 \le x^2 < \infty$

$-1 \le \sin(\frac{1}{x}) \le 1$

$-x^2 \le x^2 \sin(\frac{1}{x}) \le x^2$

$\lim_{x\to 0}(-x^2) = 0 \qquad \lim_{x\to 0} x^2 = 0$

Then $\lim_{x\to 0} x^2 \sin(\frac{1}{x})$ exists and is equal to 0.