Slope of tangent line

\[ m_{PA} = \frac{f(b) - f(a)}{b - a} \]

### § 2.2 Finding Limits Graphically and Numerically

**Def:** Suppose \( f(x) \) is defined when \( x \) is near \( a \), but not necessarily at \( a \).

We say the limit of \( f(x) \) is \( L \) as \( x \) approaches \( a \), and we write this as

\[ \lim_{x \to a} f(x) = L \]

provided we can get \( f(x) \) as close as we want to \( L \) for all \( x \) sufficiently close to \( a \), without letting \( x = a \).

**Ex 1)** \( \frac{1}{x} \) \:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \frac{1}{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \lim_{x \to 2} (4x + 5) = 11 \]

\[
\begin{array}{c|c|c|c|c|c}
\hline
x & -2.1 & -2.01 & -2.001 & -2.0001 & -2 \\
\hline
f(x) & -3.4 & -3.04 & -3.004 & -3.0004 & -3 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
\hline
x & -2 & -1.9999 & -1.999 & -1.99 & -1.9 \\
\hline
f(x) & -2.9996 & -2.996 & -2.96 & -2.6 \\
\hline
\end{array}
\]
Ex. 2 \[ \lim_{{x \to 0}} \frac{x}{\sqrt{x+1} - 1} = 2 \]

<table>
<thead>
<tr>
<th>x</th>
<th>-0.01</th>
<th>-0.001</th>
<th>0</th>
<th>0.0001</th>
<th>0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>1.99999</td>
<td>1.999999</td>
<td>2.00001</td>
<td>2.00005</td>
<td></td>
</tr>
</tbody>
</table>

Ex. 3 \[ \lim_{{x \to 0}} \frac{|x|}{x} = \text{DNE} \]

Ex. 4 \[ \lim_{{x \to 0}} \frac{1}{x} = \text{DNE} \]

Ex. 5 \[ \lim_{{x \to 0}} \frac{1}{x^2} = \text{DNE (\infty)} \]

Ex. 6 \[ \lim_{{x \to 0}} \sin \left( \frac{1}{x} \right) \]

<table>
<thead>
<tr>
<th>x</th>
<th>(\frac{2}{\pi})</th>
<th>(\frac{2}{3\pi})</th>
<th>(\frac{2}{5\pi})</th>
<th>(\frac{2}{7\pi})</th>
<th>(\frac{2}{9\pi})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin \frac{1}{x})</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>
Evaluating limits Analytically

Some basic limits:

\[ \lim_{x \to c} a = a \]

\[ y = f(x) = a \]

\[ \lim_{x \to c} x = c \]

\[ y = f(x) = x \]

Properties of limits:

1) \[ \lim_{x \to c} a \cdot f(x) = a \cdot \lim_{x \to c} f(x) = a \cdot L \] —— scalar multiple

2) \[ \lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = L \pm K \] —— sum or difference

3) \[ \lim_{x \to c} [f(x) \cdot g(x)] = \left[ \lim_{x \to c} f(x) \right] \cdot \left[ \lim_{x \to c} g(x) \right] = L \cdot K \] —— product

4) \[ \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{L}{K} \quad K \neq 0 \] —— quotient

5) \[ \lim_{x \to c} (f(x))^n = \left[ \lim_{x \to c} f(x) \right]^n = L^n \] —— power