Math 2413
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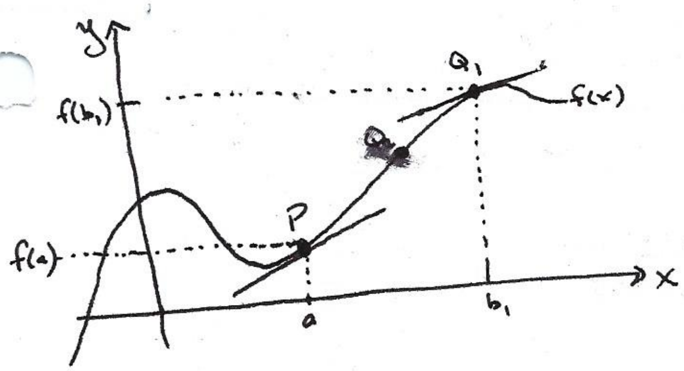
pg1

Slope of tangent line



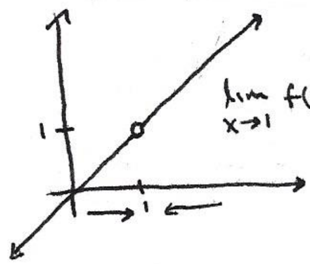
$$m_{PQ_1} = \frac{f(b_1) - f(a)}{b_1 - a}$$

§ 2.2 Finding Limits Graphically and Numerically

Def: Suppose $f(x)$ is defined when x is near a, but not necessarily at a.
We say the limit of $f(x)$ is L as x approaches a, and we write this as

$$\lim_{x \to a} f(x) = L$$

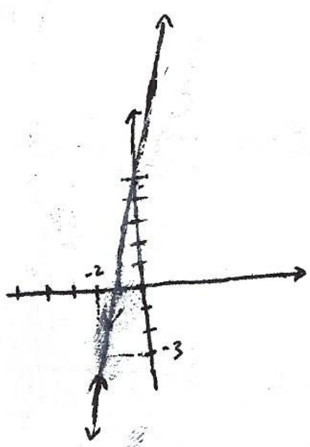
provided we can get $f(x)$ as close as we want to L for all x sufficiently close to a without letting x be a.



$$\lim_{x \to 1} f(x) = 1$$

Ex 1) $\lim_{x \to -2} (4x+5)$

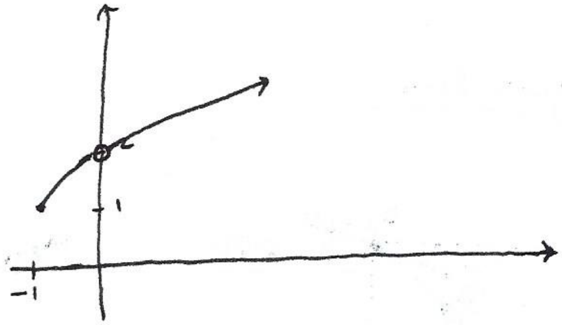
x	y
0	5
-1	1



x	-2.1	-2.01	-2.001	-2.0001	-2
f(x)	-3.4	-3.04	-3.004	-3.0004	

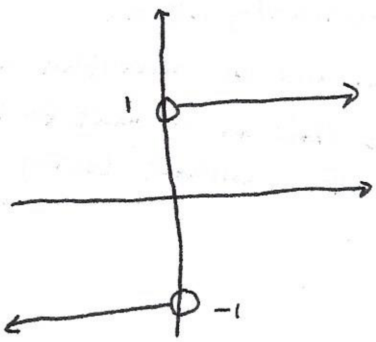
x	-2	-1.9999	-1.999	-1.99	-1.9
f(x)		-2.9996	-2.996	-2.96	-2.6

Ex. 2 $\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1} = 2$

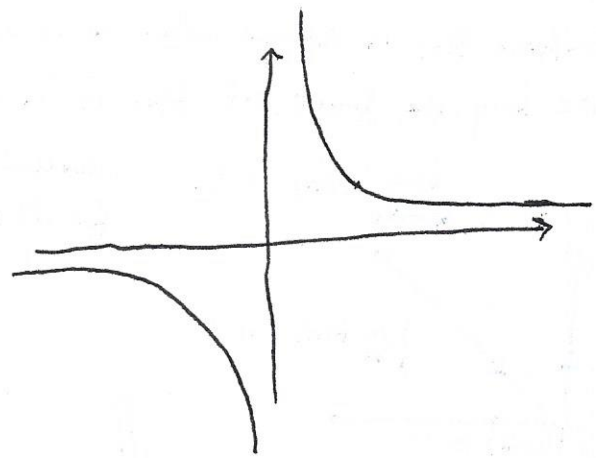


x	-0.01	-0.001	0	0.0001	0.001
f(x)	1.99499	1.99950		2.00005	2.0005

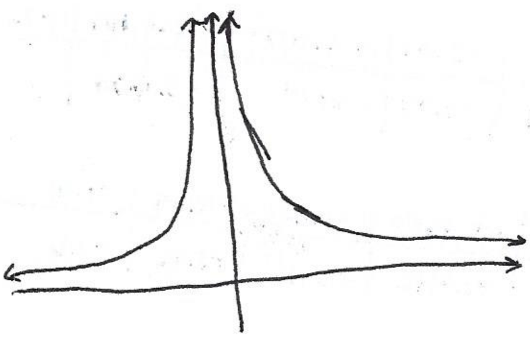
Ex 3 $\lim_{x \to 0} \frac{|x|}{x} = \text{DNE}$



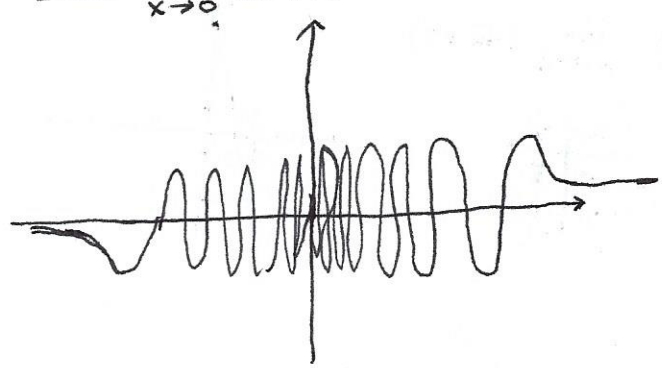
Ex 4 $\lim_{x \to 0} \frac{1}{x} = \text{DNE}$



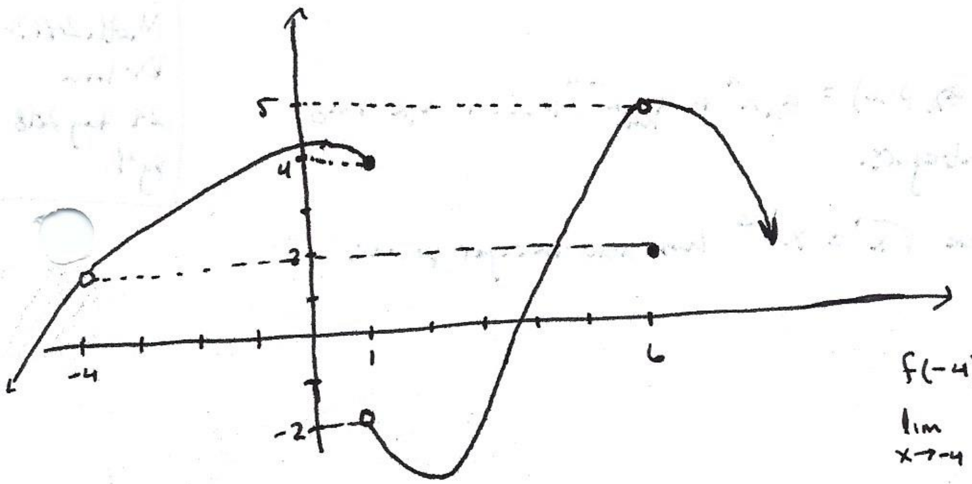
Ex 5 $\lim_{x \to 0} \frac{1}{x^2} = \text{DNE } (\infty)$



Ex 6 $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$



x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$
$\sin\frac{1}{x}$	1	-1	1	-1	1



$f(-4) = \text{DNE}$

$\lim_{x \to -4} f(x) = 2$

$f(1) = 4$

$\lim_{x \to 1} f(x) = \text{DNE}$

$f(6) = 2$

$\lim_{x \to 6} f(x) = 5$

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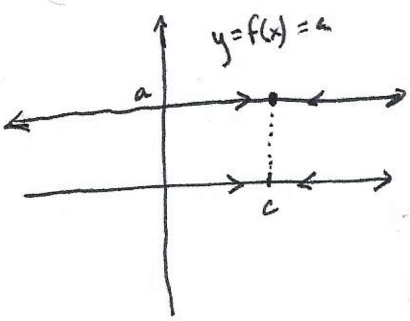
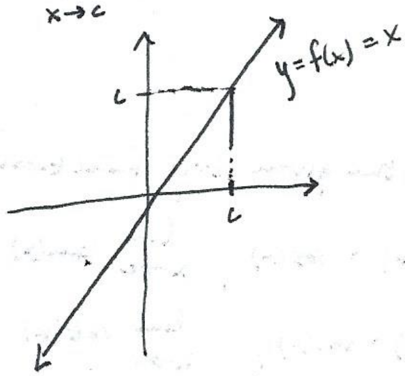
Evaluating Limits Analytically

- Some basic limits:

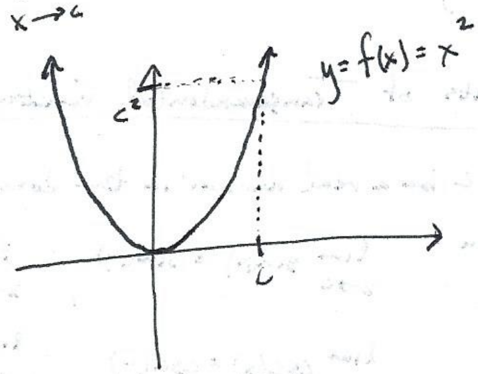
$\lim_{x \to c} a = a$



$\lim_{x \to c} x = c$



$\lim_{x \to c} x^n = c^n$



Properties of Limits:

1) $\lim_{x \to c} a \cdot f(x) = a \cdot \lim_{x \to c} f(x) = a \cdot L$ ---- scalar multiple

2) $\lim_{x \to c} \left[f(x) \pm g(x)\right] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = L \pm K$ -- sum or difference

3) $\lim_{x \to c} \left[f(x) \cdot g(x)\right] = \left[\lim_{x \to c} f(x)\right] \cdot \left[\lim_{x \to c} g(x)\right] = L \cdot K$ ---- product

4) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{L}{K}, \; K \neq 0$ ---- quotient

5) $\lim_{x \to c} \left(f(x)\right)^n = \left[\lim_{x \to c} f(x)\right]^n = L^n$ ---- power