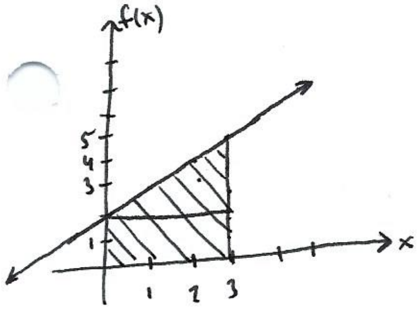
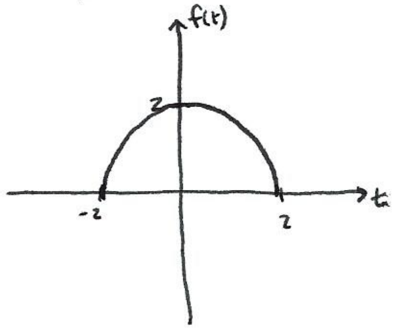
Ex. $\int_0^3 (x+2)\,dx = \frac{1}{2}\cdot 3\cdot 3 + 3\cdot 2 = \frac{9}{2} + 6 = \frac{21}{2} = 10.5$



Ex $\int_{-2}^{2} (4-t^2)^{1/2}\,dt = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi 2^2 = 2\pi$



Def. If f is defined at a, then

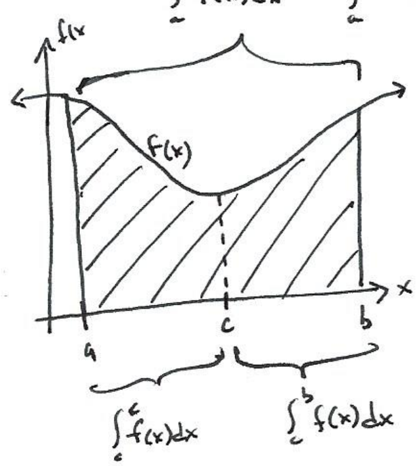
$$\int_a^a f(x)\,dx = 0$$

Def. If f is integrable on $[a,b]$, then

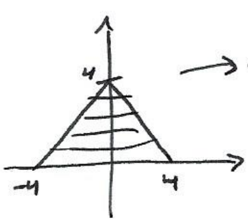
$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

Thm. If f is integrable on the three closed intervals determined by $a, b,$ and c, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

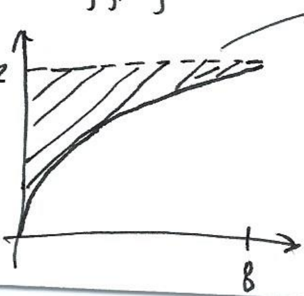


Ex $f(x) = 4 - |x|$



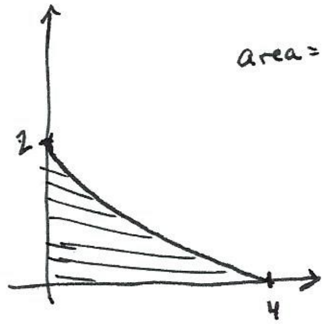
$\rightarrow$ area $= \int_{-4}^{4} (4-|x|)\,dx = 2\int_0^4 (4-|x|)\,dx = 2\int_{-4}^{0} (4-|x|)\,dx$

Ex $g(y) = y^3$



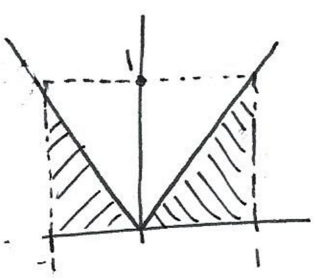
$\rightarrow$ area $= \int_0^2 y^3\,dy$

Ex $x = (y-2)^2 \mapsto y = 2 \pm \sqrt{x} \mapsto y = 2 - \sqrt{x}$



$$\text{area} = \int_0^2 (y-2)^2 \, dy = \int_0^4 (2 - \sqrt{x}) \, dx$$

Ex $\int_{-1}^{1} |x| \, dx = \int_{-1}^{0} (-x) \, dx + \int_0^1 (x) \, dx = -\frac{x^2}{2}\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_0^1 = \left(0 - -\frac{1}{2}\right) + \left(\frac{1}{2} - 0\right) = 1$



$$\text{area} = 2 \cdot \frac{1}{2} \cdot 1 \cdot 1 = 1$$

Thm: If f and g are integrable on $[a,b]$ and k is a constant, then the functions kf and $f \pm g$ are integrable on $[a,b]$, and

$$\int_a^b k f(x) \, dx = k \int_a^b f(x) \, dx$$

$$\int_a^b [f(x) \pm g(x)] \, dx = \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx$$

Ex. Given $\int_0^5 f(x) \, dx = 10$, $\int_5^7 f(x) \, dx = 3$, and $\int_5^7 g(x) \, dx = 2$, evaluate

a) $\int_7^0 f(x) \, dx = -\int_0^7 f(x) \, dx = -\left[\int_0^5 f(x) \, dx + \int_5^7 f(x) \, dx\right] = -(10 + 3) = -13$

b) $\int_5^5 f(x) \, dx = 0$

c) $\int_5^7 [f(x) - 3g(x) + 1] \, dx = \int_5^7 f(x) \, dx - \int_5^7 3g(x) \, dx + \int_5^7 (1) \, dx = 3 - 3(2) + 2 \cdot 1 = -1$

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Ex (cont)

d) $\int_5^7 f(x)\,g(x)\,dx$ cannot be determined

e) $\int_5^7 (f(x))^3\,dx$ cannot be determined

f) $\int_5^7 \frac{f(x)}{g(x)}\,dx$ cannot be determined

Ex. Use the given table to estimate $\int_0^6 f(x)\,dx$. use three equal subintervals and

a) left endpoints
b) right endpoints
c) midpoints

when f is increasing function how does each estimate compare w/ actual value?

x	0	1	2	3	4	5	6
f(x)	-6	0	8	18	30	50	80

$\Delta x = \frac{6-0}{3} = 2$

a) $\int_0^6 f(x)\,dx$

$= f(0)\cdot 2 + f(2)\cdot 2 + f(4)\cdot 2$

$= -6\cdot 2 + 8\cdot 2 + 30\cdot 2 = -12 + 16 + 60 = 64$

b) $\int_0^6 f(x)\,dx = f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2$

$= 8\cdot 2 + 30\cdot 2 + 80\cdot 2 = 16 + 60 + 160 = 236$

c) $\int_0^6 f(x)\,dx = f(1)\cdot 2 + f(3)\cdot 2 + f(5)\cdot 2$

$= 0\cdot 2 + 18\cdot 2 + 50\cdot 2 = 136$