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Dr. Liu
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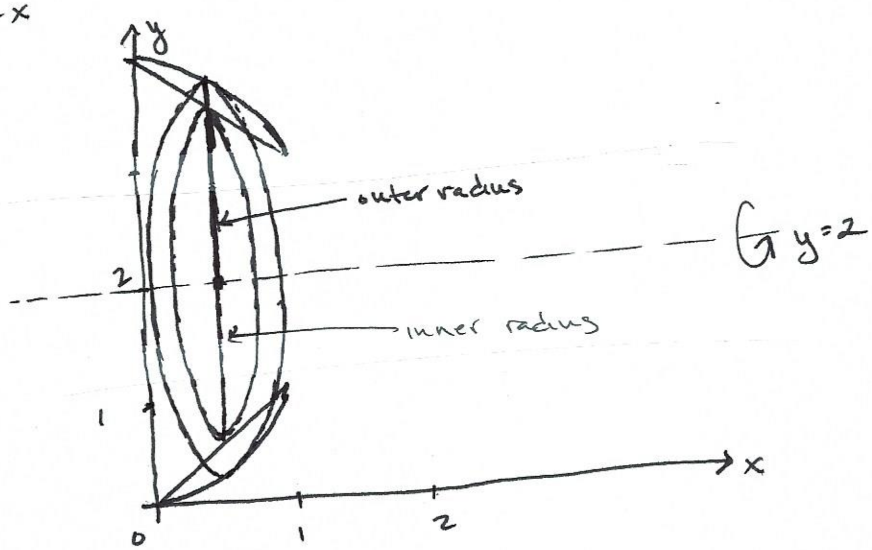
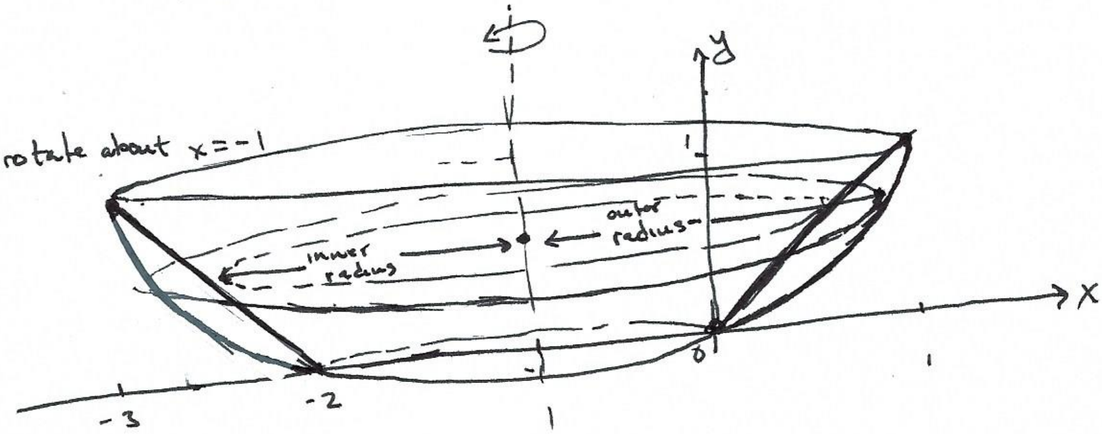
Ex. $f(x) = x^2$, $x \in [0,1]$, rotate about $y = 2$
$g(x) = x$



$$V = \pi \int_0^1 \left[\underbrace{\left(2 - x^2\right)^2}_{\text{outer radius}} - \underbrace{\left(2 - x\right)^2}_{\text{inner radius}} \right] dx$$

Ex. $f(y) = y$ rotate about $x = -1$
$g(y) = \sqrt{y}$



$$V = \pi \int_0^1 \left[\underbrace{\left(\sqrt{y} - (-1)\right)^2}_{\text{outer radius}} - \underbrace{\left(y - (-1)\right)^2}_{\text{inner radius}} \right] dx$$

$$= \pi \int_0^1 \left[\left(\sqrt{y} + 1\right)^2 - \left(y + 1\right)^2 \right] dx$$

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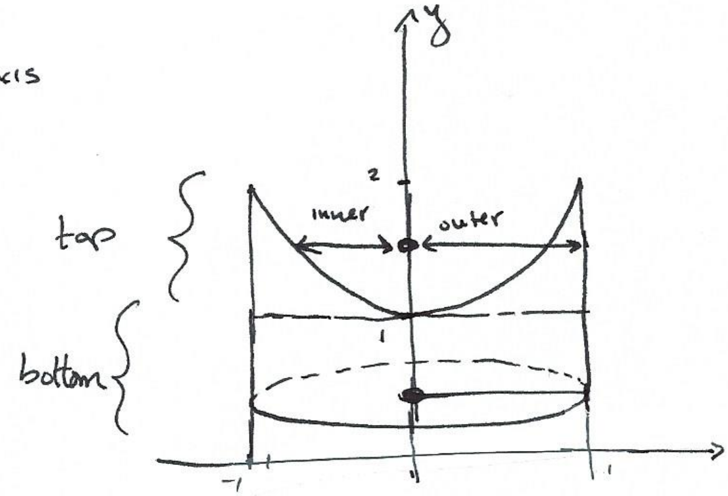
Ex. $f(x) = x^2+1$ rotate abut y-axis
$y=0$
$x=0$
$x=1$

$V_{cyl} = \pi r^2 h$



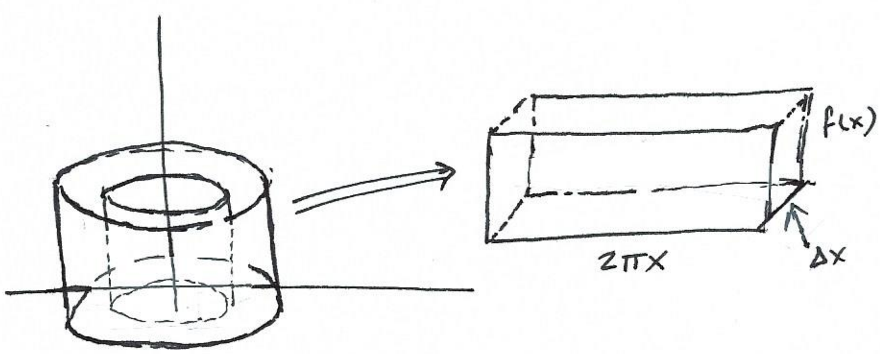
$$V = V_{bottom} + V_{top} = \underbrace{\pi\int_0^1 (1)^2\,dy}_{V_{bottom}} + \pi\int_1^2 \Big[\underbrace{(1)^2}_{outer} - \underbrace{(\sqrt{y-1})^2}_{inner}\Big]\,dy$$

$$= \pi\int_0^1 dy + \pi\int_1^2 \big[1-(y-1)\big]\,dy = \frac{3\pi}{2}$$

Volume: Shell Method

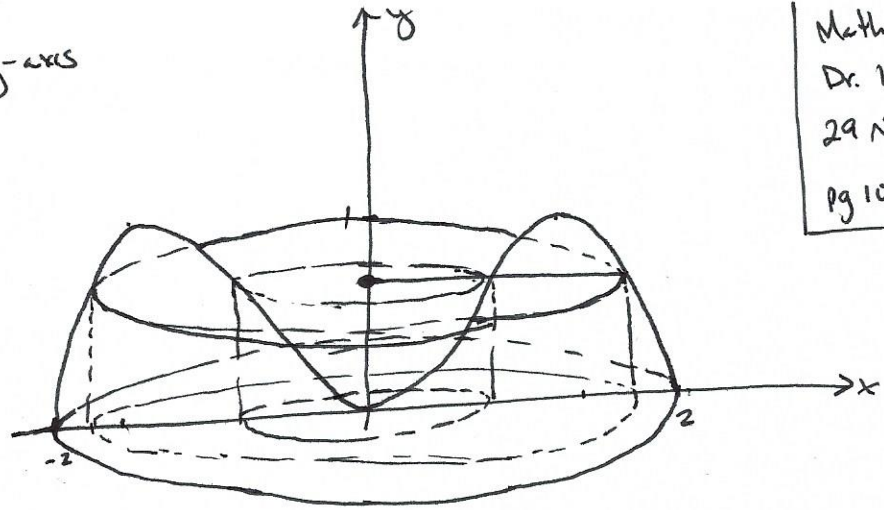
Horizontal axis of Revolution : $V = \int_c^d 2\pi r(y) f(y)\,dy$

Vertical " " : $V = \int_a^b 2\pi r(x) f(x)\,dx$



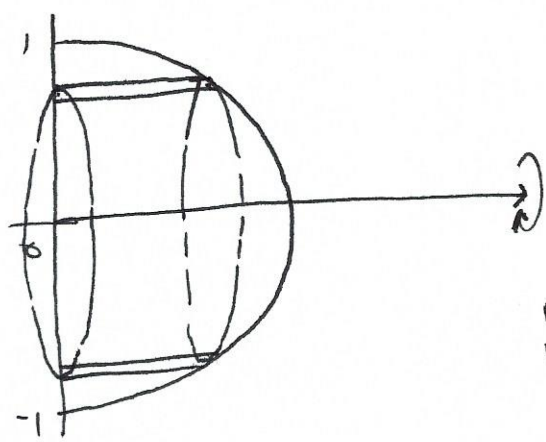
$V = 2\pi x f(x)\,\Delta x$

Ex. $f(x) = 2x^2 - x^3$, rotate about y-axis



$$V = 2\pi \int_0^2 x\left(2x^2 - x^3\right) dx$$

Ex. Rotate $x = e^{-y^2}$, $x = 0$, $0 \le y \le 1$ about the x-axis



$$V = 2\pi \int_0^1 y e^{-y^2} dy = -\pi \int_0^{-1} e^u du = -\pi e^u \Big|_0^{-1} = -\pi(e^{-1} - 1) = \pi\left(1 - \frac{1}{e}\right)$$

let $u = -y^2$ $\quad y = 0 \to u = 0$
$du = -2y\,dy$ $\quad y = 1 \to u = -1$

radius $\to y$
height $\to e^{-y^2}$

Ex $y = x^3 + x + 1$, $y = 1$, $x = 1$
rotate about line $x = 2$

$$V = 2\pi \int_0^1 \underset{\substack{\uparrow \\ \text{radius}}}{(2-x)} \underset{\substack{\uparrow \\ \text{height}}}{(x^3 + x + 1 - 1)} dx = \ldots = \frac{29\pi}{15}$$