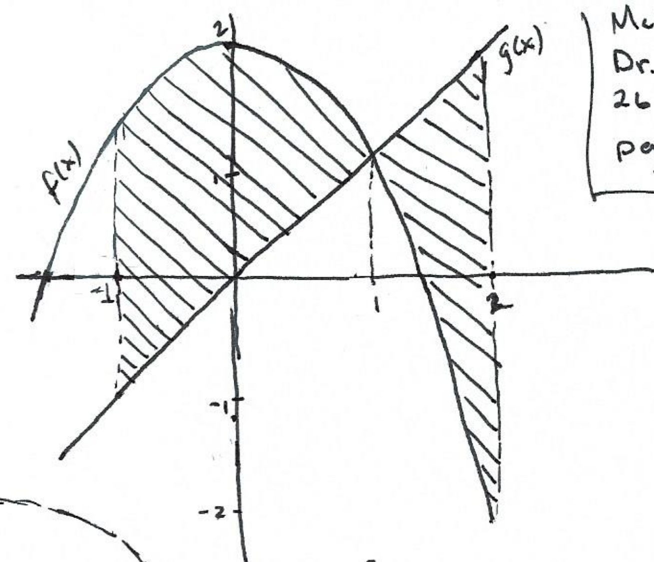
__Ex.__ $f(x) = 2 - x^2$, $g(x) = x$ on $[-1, 2]$



$$\text{Area} = \int_{-1}^{1} (2 - x^2 - x)\,dx + \int_{1}^{2} (x - 2 + x^2)\,dx$$

$$= \dots = \frac{31}{6}$$

$2 - x^2 = x$

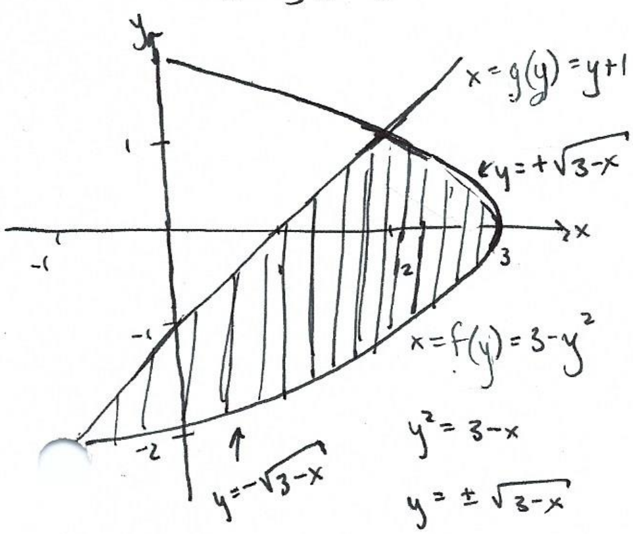
$-x^2 - x + 2 = 0$

$(-x+1)(x+2) = 0$

$x = 1, -2$

$\rightarrow x = 1$

__Ex.__ $f(y) = 3 - y^2$, $g(y) = y + 1$



$3 - y^2 = y + 1$

$y^2 + y - 2 = 0$

$(y-1)(y+2)$

$y = 1, -2$

$y = -2 \longrightarrow x = -1$

$y = 1 \longrightarrow x = 2$

$y^2 = 3 - x$

$y = \pm\sqrt{3-x}$

$$\text{Area} = \int_{-1}^{2} \left(x - 1 - (-\sqrt{3-x})\right) dx + \int_{2}^{3} \left[(\sqrt{3-x}) - (-\sqrt{3-x})\right] dx$$

$$= \int_{-2}^{1} \left[(3 - y^2) - (y+1)\right] dy$$

$$= \frac{9}{2}$$

Volume

Disk Method

Def. For cross sections of area $A(x)$ taken perpendicular to x-axis,

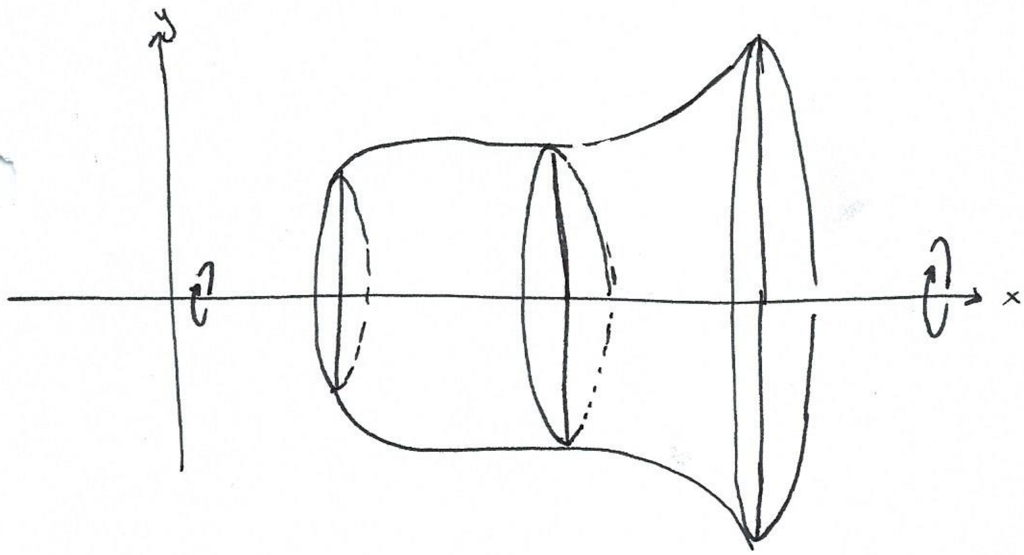
the volume of a solid S is

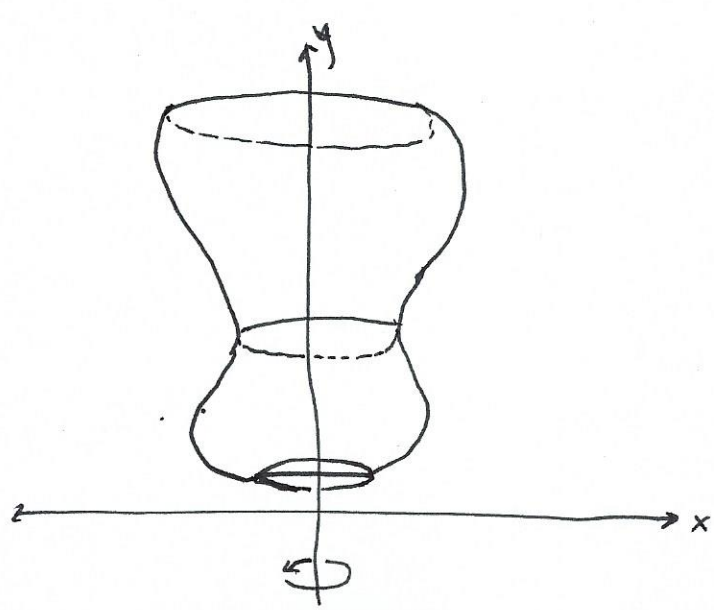
$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\,\Delta x = \int_a^b A(x)\,dx$$

To find the volume of a solid of revolution with the disk method,

Horizontal axis: $V = \int_a^b \pi \cdot [R(x)]^2\,dx$

Vertical axis: $V = \int_c^d \pi \cdot [R(y)]^2\,dy$





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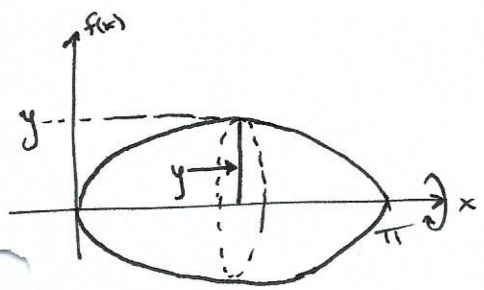
Consider a region bounded by an outer radius $R(x)$ and an inner radius $r(x)$. To find the volume of a solid of revolution with the washer method,

Horizontal axis: $$V = \int_a^b \pi\left[R(x)^2 - r(x)^2\right]dx$$

Vertical axis : $$V = \int_c^d \pi\left[R(y)^2 - r(y)^2\right]dy$$

Ex. Find the volume of the solid formed by revolving $\underbrace{f(x) = \sqrt{\sin(x)}}_{y}$ about the x-axis from $x = 0$ to $x = \pi$

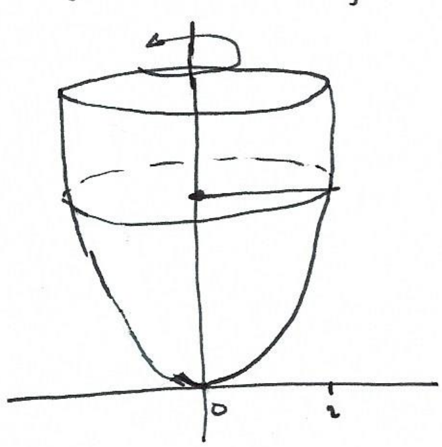


$$V = \int_0^\pi \pi \cdot \left(\sqrt{\sin(x)}\right)^2 dx$$

$$= \pi \int_0^\pi \sin(x)dx = \pi\left[-\cos(x)\right]_0^\pi$$

$$= \pi\left[-\cos(\pi) - (-\cos(0))\right] = 2\pi$$

Ex. $y = x^3$, $x \in [0,2]$, y-axis



$$V = \int_0^8 \pi\left(y^{1/3}\right)^2 dy = \pi\int_0^8 y^{2/3} dy = \pi \frac{y^{5/3}}{5/3}\Bigg|_0^8$$

$$= \frac{3\pi}{5} 8^{5/3}$$