\[ f(x) = 2 - x^2, \quad g(x) = x \quad \text{on} \quad [-1, 2] \]

\[
\text{Area} = \int_{-1}^{2} (2 - x^2 - x) \, dx + \int_{1}^{2} (x - 2 + x^2) \, dx
\]

\[ = \ldots = \frac{3}{16} \]

\[ f(y) = 3 - y^2, \quad g(y) = y + 1 \]

\[ x = g(y) = y + 1 \]

\[ x = f(y) = 3 - y^2 \]

\[ y^2 = 3 - x \]

\[ y = \pm \sqrt{3-x} \]

\[
\text{Area} = \int_{-1}^{2} (x - 1 - (-\sqrt{3-x})) \, dx + \int_{3}^{2} \left[ (\sqrt{3-x}) - (-\sqrt{3-x}) \right] \, dx
\]

\[ = \int_{-2}^{1} \left[ (3-y^2) - (y+1) \right] \, dy \]

\[ = \frac{9}{2} \]
Volume

Disk Method

Def. For cross sections of area \( A(x) \) taken perpendicular to \( x \)-axis,

the volume of a solid \( S \) is

\[
V = \lim_{n \to \infty} \sum_{i=1}^{n} A(y_i) \Delta x = \int_{a}^{b} A(x) \, dx
\]

To find the volume of a solid of revolution with the disk method,

- **Horizontal axis:** \( V = \int_{a}^{b} \pi \left[ R(x) \right]^2 \, dx \)
- **Vertical axis:** \( V = \int_{c}^{d} \pi \left[ R(y) \right]^2 \, dy \)
Consider a region bounded by an outer radius $R(x)$ and an inner radius $r(x)$. To find the volume of a solid of revolution with the washer method,

**Horizontal axis:** \[ V = \int_a^b \pi \left[ R(x)^2 - r(x)^2 \right] \, dx \]

**Vertical axis:** \[ V = \int_c^d \pi \left[ R(y)^2 - r(y)^2 \right] \, dy \]

**Ex.** Find the volume of the solid formed by revolving $f(x) = (\sin x)^2$ about the $x$-axis from $x = 0$ to $x = \pi$.

\[
V = \int_0^\pi \pi \left( (\sin x)^2 \right)^2 \, dx \\
= \pi \int_0^\pi (\sin x)^2 \, dx = \pi \left[ -\cos x \right]_0^\pi \\
= \pi \left[ -\cos(\pi) - (-\cos(0)) \right] = 2\pi
\]

**Ex.** $y = x^2$, $x \in [0,2]$, $y$-axis.

\[
V = \int_0^8 \pi \left( \sqrt[3]{y} \right)^2 \, dy = \pi \int_0^8 \frac{y^{2/3}}{2/3} \, dy = \pi \left[ \frac{3}{8} y^{5/3} \right]_0^8 \\
= \frac{3\pi}{5} \times 8^{5/3} = \frac{3\pi}{5} \times 8^{5/3}
\]