Math 2413
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Ex Evaluate the sum $\sum_{i=1}^{n} \frac{i+1}{n^2}$ for $n=10$ and $n=100$.

$$n=10: \sum_{i=1}^{10} \frac{i+1}{100} = \frac{1}{100}\sum_{i=1}^{10}(i+1) = \frac{1}{100}\sum_{i=1}^{10} i + \frac{1}{100}\sum_{i=1}^{10}(1) = \frac{1}{100}\,\frac{10(10+1)}{2} + \frac{1}{100}(10)$$

$$= \frac{55}{100} + \frac{10}{100} = \frac{65}{100} = \frac{13}{20}$$

$$n=100: \sum_{i=1}^{100} \frac{i+1}{100^2} = \frac{1}{100^2}\sum(i+1) = \frac{1}{100^2}\left[\frac{100(100+1)}{2} + 100\right] = \frac{1}{100}\left[\frac{101}{2} + 1\right] = \frac{103}{200}$$

Ex Evaluate $\sum_{k=1}^{n} \frac{6k(k-1)}{n^3}$ for $n=10$ and $n=1000$.

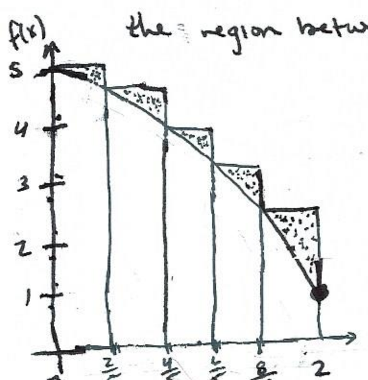
$$\sum_{k=1}^{n} \frac{6k(k-1)}{n^3} = \frac{6}{n^3}\sum_{k=1}^{n}(k^2-k) = \frac{6}{n^3}\left[\sum_{k=1}^{n}k^2 - \sum_{k=1}^{n}k\right] = \frac{6}{n^3}\left[\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}\right]$$

$$= \frac{n+1}{n^2}\left[(2n+1)-3\right] = \frac{n+1}{n^2}(2n-2) = \frac{2}{n^2}(n+1)(n-1)$$

$$n=10: \sum_{k=1}^{n}\frac{6k(k-1)}{n^3} = \frac{2}{10^2}(11)(9) = \frac{99}{50}$$

$$n=1000: \sum_{k=1}^{n}\frac{6k(k-1)}{n^3} = \frac{2}{1000^2}(1001)(999)$$

Ex. Use left and right endpoints and five rectangles to find two approximations of the area of the region between the graph of $f(x) = -x^2+5$ and the x-axis over $[0,2]$. Sketch the region.



overestimate

$$\frac{2}{5}\cdot f(0) + \frac{2}{5}\cdot f\left(\frac{2}{5}\right) + \frac{2}{5}\cdot f\left(\frac{4}{5}\right) + \frac{2}{5}\cdot f\left(\frac{6}{5}\right) + \frac{2}{5}\cdot f\left(\frac{8}{5}\right) = \frac{2}{5}\left[f(0) + f\left(\frac{2}{5}\right) + f\left(\frac{4}{5}\right) + f\left(\frac{6}{5}\right) + f\left(\frac{8}{5}\right)\right]$$

Left end points are $0 + (i-1)\frac{2}{5}$ for $i = 1,2,3,4,5$, so

area is $\sum_{i=1}^{5} \frac{2}{5} f\left((i-1)\frac{2}{5}\right) = \frac{2}{5}\sum_{i=1}^{5}\left(-\frac{4}{25}(i-1)^2 + 5\right) = \frac{2}{5}\left[\sum_{i=1}^{5} -\frac{4}{25}(i-1)^2 + \sum_{i=1}^{5} 5\right]$

$$= \frac{2}{5}\left[\frac{-4}{25}\sum_{i=1}^{5}(i^2-2i+1) + 5\cdot 5\right] = \frac{2}{5}\left[\frac{-4}{25}\left(\frac{5\cdot 6\cdot 11}{6} - 2\cdot\frac{5\cdot 6}{2} + 1\cdot 5\right) + 25\right]$$

$$= \dots = \frac{202}{25}$$

$\boxed{}$ = error

right endpoints: $0 + i\cdot\frac{2}{5} = \frac{2}{5}i$, $i = 1,2,3,4,5$

area is $\sum_{i=1}^{5}\frac{2}{5} f\left(\frac{2}{5}i\right) = \frac{2}{5}\sum_{i=1}^{5}\left[-\left(\frac{2}{5}i\right)^2 + 5\right] = \frac{2}{5}\sum_{i=1}^{5}\left(\frac{-4}{25}i^2 + 5\right)$

$$= \frac{2}{5}\left(\frac{-4}{25}\,\frac{5\cdot 6\cdot 11}{6} + 5\cdot 5\right) = \dots = \frac{162}{25}$$

underestimate