Math 2413
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EX $\int_1^2 e^{1-x}\,dx = \int_0^{-1} -e^u\,du = -e^u\Big|_0^{-1} = -e^{-1} - -e^0 = \frac{-1}{e} + 1$

let $u = 1-x$ $\quad x = 1 \to u = 0$

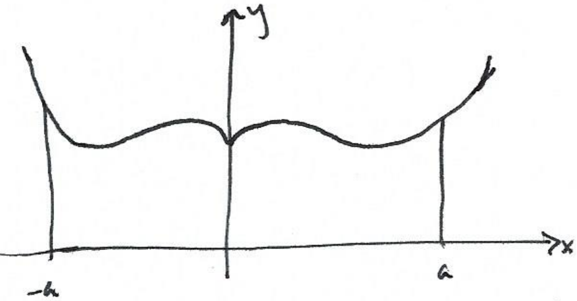
$du = -dx$ $\quad x = 2 \to u = -1$

$* \int e^{1-x}\,dx = \int e^{-u}(-du) = -e^u + C = -e^{1-x} + C$

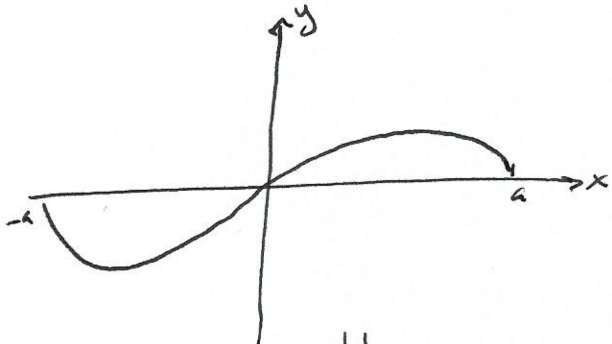
Thm. Let $f$ be integrable on $[-a, a]$

1) if $f$ is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

2) if $f$ is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$.




even




odd

EX $\int_{-2}^{2} x^4\,dx = \frac{x^5}{5}\Big|_{-2}^{2} = \frac{2^5}{5} - \frac{(-2)^5}{5} = \frac{32}{5} + \frac{32}{5} = \frac{64}{5}$

Recall:
$f(x)$ is even if $f(-x) = f(x)$
$f(x)$ is odd iff $f(-x) = -f(x)$

or

$f(x) = x^4$ is even because $f(-x) = (-x)^4 = x^4 = f(x)$, so

$\int_{-2}^{2} x^4\,dx = 2\int_0^2 x^4\,dx = 2\cdot\frac{x^5}{5}\Big|_0^2 = 2\cdot\frac{2^5}{5} = 2\cdot\frac{32}{5} = \frac{64}{5}$

EX. $\int_{-\pi/2}^{\pi/2} \left(\sin^3(x)\cos(x) + \sin(x)\cos(x)\right)dx = \int_{-\pi/2}^{\pi/2} \sin^3(x)\cos(x)\,dx + \int_{-\pi/2}^{\pi/2} \sin(x)\cos(x)\,dx = \ldots$

let $u = \sin(x)$
$du = \ldots$
$\vdots$

let $u = \sin(x)$
$\vdots$

OR

$f(-x) = [\sin(-x)]^3\cos(-x) + \sin(-x)\cos(-x) = (-\sin(x))^3\cos(x) - \sin(x)\cos(x) = -\sin^3(x)\cos(x) - \sin(x)\cos(x) = -f(x)$

So $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\sin^3(x)\cos(x) + \sin(x)\cos(x)\right]dx = 0$

# Natural Log Function: Integration



**Thm.** $\int \frac{1}{x}dx = \ln|x| + C$

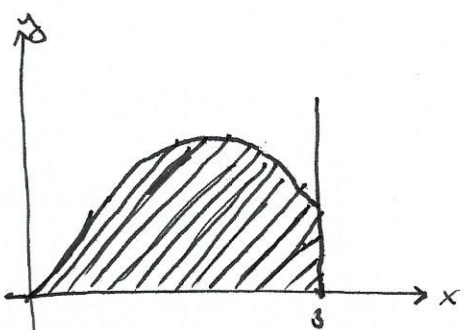
$\int \tan(x)\,dx = -\ln|\cos(x)| + C$

$\int \cot(x)\,dx = \ln|\sin(x)| + C$

$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$

$\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)| + C$

**Ex.** Find the area of the region bounded by $y = \frac{x}{x^2+1}$, the x-axis, and the line $x=3$.




$$\int_0^3 \frac{x}{x^2+1}\,dx = \int_1^{10} \frac{\frac{1}{2}du}{u} = \frac{1}{2}\ln|u|\Big|_1^{10} = \frac{1}{2}\ln|10| - \frac{1}{2}\ln|1| = \frac{1}{2}\ln(10)$$

let $u = x^2+1$ $\quad x=0 \to u=1$

$du = 2x\,dx$ $\quad x=3 \to u=10$

**Ex** $\int \frac{\sec^2(x)}{\tan(x)}dx = \int \frac{du}{u} = \ln|u| + C = \ln|\tan(x)| + C$

let $u = \tan(x)$

$du = \sec^2(x)dx$

OR

$\int \frac{\sec^2(x)}{\tan(x)}dx = \int \frac{1}{\cos^2(x)} \cdot \frac{\cos(x)}{\sin(x)}dx = \int \frac{du}{\cos(x)\sin(x)} = ?$ we cannot compute this directly.

**Ex.** $\int \frac{x^2+x+1}{x^2+1} = \int \left(1 + \frac{x}{x^2+1}\right)dx = \int dx + \int \frac{x}{x^2+1}dx$

$= \int dx + \int \frac{\frac{1}{2}du}{u}$

$= x + \frac{1}{2}\ln|u| + C$

$= x + \frac{1}{2}\ln|x^2+1| + C$

$= x + \frac{1}{2}\ln(x^2+1) + C$

$$\begin{array}{r|l} & 1 \\ x^2+1 & x^2+x+1 \\ & x^2+0+1 \\ \hline & x \end{array}$$