Math 2413
Dr. Liu
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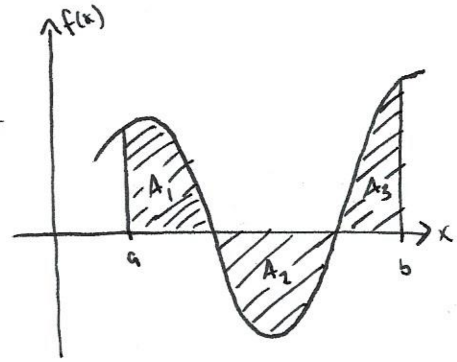
Note:



A definite integral can be interpreted as a <u>net area</u>, that is, a difference of areas:

$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$

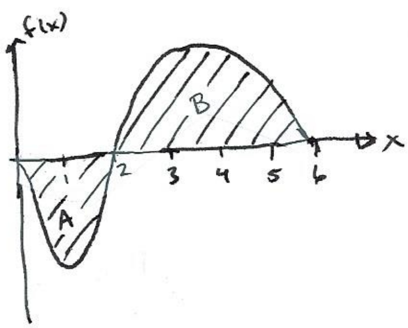
Another way to illustrate the Net Change Theorem is to examine velocity of a particle moving along a straight line, where $s(t)$ is the position at time t. Then its velocity is $v(t) = s'(t)$ and $\int_a^b v(t)\,dt = s(b) - s(a)$.

This definite integral gives net change in position, or <u>displacement</u>, of the particle. We must consider intervals where $v(t) \le 0$ (particle moves left) and where $v(t) \ge 0$ (or at rest) (particle at rest or moving right). To calculate total dist traveled, integrate $|v(t)|$.

So, on $[a,b]$, the <u>displacement</u> is $\int_a^b v(t)\,dt = A_1 - A_2 + A_3$

and the <u>total distance traveled</u> is $\int_a^b |v(t)|\,dt = A_1 + A_2 + A_3$

Ex



$\int_0^2 f(x)\,dx = -1.5$

Region A has area 1.5, and $\int_0^6 f(x)\,dx = 3.5$.

a) $\int_0^2 (-2f(x))\,dx = -2\int_0^2 f(x)\,dx = -2\cdot(-1.5) = 3$

b) $\int_2^6 f(x)\,dx = \int_0^6 f(x)\,dx - \int_0^2 f(x)\,dx = 3.5 - (-1.5) = 5$

c) $\int_0^6 |f(x)|\,dx = A + B = 1.5 + 5 = 6.5$

d) $\int_0^6 (2 + f(x))\,dx = 2\int_0^6 dx + \int_0^6 f(x)\,dx = 2x\Big|_0^6 + 3.5 = 12 + 3.5 = 15.5$

e) average of f over $[0,6] = \frac{1}{6-0}\int_0^6 f(x)\,dx = \frac{3.5}{6} = \frac{7}{12}$

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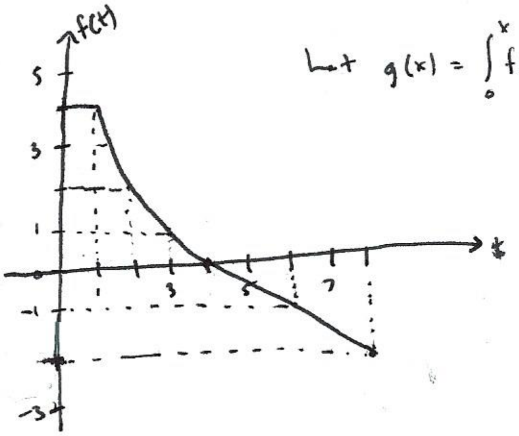
Ex



Let $g(x) = \int_0^x f(t)\,dt$

a) estimate $g(0)$, $g(4)$, $g(8)$

$$g(0) \approx \int_0^0 f(t)\,dt = 0$$

$$g(4) = \int_0^4 f(t)\,dt \approx 1\cdot 4 + 1\cdot 4 + 1\cdot 2 + 1\cdot 1 = 11 \quad \leftarrow \text{left enpoints}$$

$$\approx 1\cdot 4 + 1\cdot 2 + 1\cdot 1 + 1\cdot 0 = 7 \quad \leftarrow \text{right endpoints}$$

$$\approx 9 \quad \leftarrow \text{count squares}$$

$$g(8) = \int_0^8 f(t)\,dt \approx 9 - \tfrac{1}{2}(8-4)(2) = 5$$

b) largest open interval where g increasing:

$(0,4)$

largest open interval where g decreasing:

$(4,8)$

c) extrema of g:

local max @ $x=4$

local min @ $x=0$

Ex The velocity (in feet per second) of a particle moving along a line is $v(t) = t^3 - 10t^2 + 29t - 20$, with t in seconds.

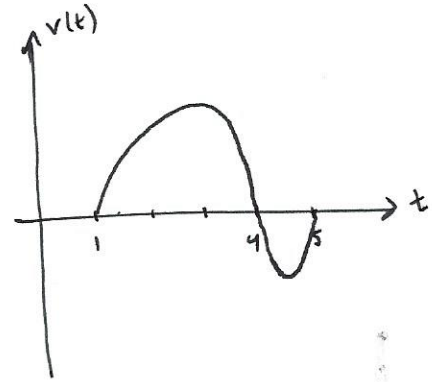
a) displacement of particle on time interval $[1,5]$

$$\int_0^5 v(t)\,dt = \int_0^5 \left(t^3 - 10t^2 + 29t - 20\right)dt = \left[\frac{t^4}{4} - \frac{10t^3}{3} + \frac{29t^2}{2} - 20t\right]_1^5$$

$$= \frac{5^4}{4} - \frac{10\cdot 5^3}{3} - \frac{29\cdot 5^2}{2} - 20\cdot 5 - \left(\frac{1}{4} - \frac{10}{3} + \frac{29}{2} - 20\right)$$

$$= \ldots = \frac{32}{3}$$

b) total distance traveled by particle on $[1,5]$



$$\int_1^5 |v(t)|\,dt = \int_1^4 v(t) + \int_4^5 -v(t)\,dt$$

$$= \left[\frac{t^4}{4} - \frac{10t^3}{3} + \frac{29t^2}{2} - 20t\right]_1^4 - \left[\frac{t^4}{4} - \frac{10t^3}{3} + \frac{29t^2}{2} - 20t\right]_4^5$$

$$= \ldots = \frac{71}{6}$$