Fundamental Theorem of Calculus

Thm. If f is continuous on $[a,b]$ and F is an antiderivative of f on $[a,b]$,

then $$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(x)\Big|_a^b = F(b) - F(a)$$

Note: $$\int_a^b f(x)\,dx = \Big[F(x)+C\Big]_a^b = \Big[F(b)+C\Big] - \Big[F(a)+C\Big] = F(b) - F(a),$$

so we do not need to include the constant of integration C.

Ex. Evaluate each definite integral.

a) $\displaystyle\int_0^2 |2x-1|\,dx$

*note: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$, so $|2x-1| = \begin{cases} 2x-1, & 2x-1 \geq 0 \\ -2x+1, & 2x-1 < 0 \end{cases} = \begin{cases} 2x-1, & x \geq \frac{1}{2} \\ -2x+1, & x < \frac{1}{2} \end{cases}$

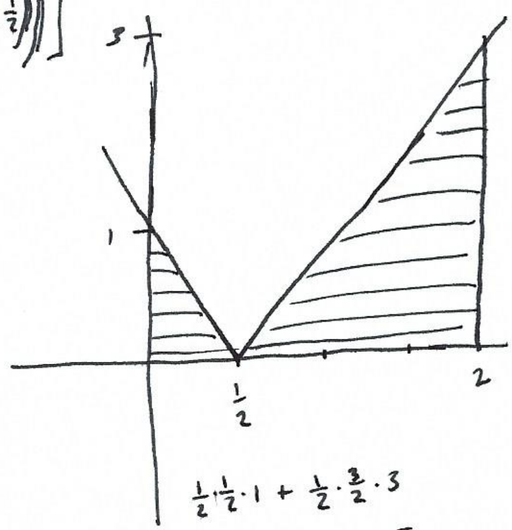
$$\int_0^2 |2x-1|\,dx = \int_0^{1/2} (-2x+1)\,dx + \int_{1/2}^2 (2x-1)\,dx$$

$$= \left[-2\cdot\frac{x^2}{2} + x\right]_0^{1/2} + \left[2\cdot\frac{x^2}{2} - x\right]_{1/2}^2$$

$$= \left[-\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right) - \left(-(0)^2 + (0)\right)\right] + \left[(2)^2 - (2) - \left(\left(\tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)\right)\right]$$

$$= \left[-\tfrac{1}{4} + \tfrac{1}{2}\right] + \left[4 - 2 - \tfrac{1}{4} + \tfrac{1}{2}\right]$$

$$= \left[\tfrac{1}{4}\right] + \left[2 + \tfrac{1}{4}\right] = 2 + \tfrac{1}{2} = \tfrac{5}{2}$$

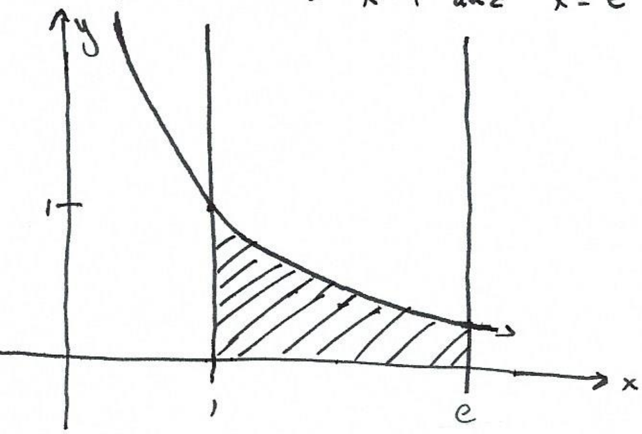


$$\tfrac{1}{2}\cdot\tfrac{1}{2}\cdot 1 + \tfrac{1}{2}\cdot\tfrac{3}{2}\cdot 3$$
$$= \tfrac{1}{4} + \tfrac{9}{4} = \tfrac{10}{4} = \tfrac{5}{2}$$

Ex. (cont)

b) $\int_0^{\pi/4} \sec^2(x)\,dx = \Big[\tan(x)\Big]_0^{\pi/4} = \tan(\frac{\pi}{4}) - \tan(0) = 1 - 0 = 1$

Ex. Find the area of the region bounded by the graph of $y = \frac{1}{x}$, the x-axis, and the vertical lines $x = 1$ and $x = e$



$$\int_1^e \frac{1}{x}\,dx = \Big[\ln|x|\Big]_1^e = \ln|e| - \ln|1| = 1 - 0 = 1$$

Mean Value Theorem for Integrals

Thm If f continuous on $[a,b]$, then there exists $c \in [a,b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$