Math 2413
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Def In many applications, enough information is given to determine a particular solution. We need only the value of $y = F(x)$ for one value of x. This information is called an initial condition.

Ex Find the particular solution that satisfies the differential equation and the initial condition.

1) $f'(x) = e^x$, $f(0) = 3$

$f(x) = \int f'(x)\,dx = \int e^x\,dx = e^x + C$

Since $f(0) = 3$, $e^0 + C = 1 + C = 3$

$\rightarrow C = 2$

So $f(x) = e^x + 2$

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2) $f''(x) = 2$, $f'(2) = 5$, $f(2) = 10$

$f'(x) = \int f''(x)\,dx = \int 2\,dx = 2x + C_1$

$f'(2) = 2(2) + C_1 = 4 + C_1 = 5 \rightarrow C_1 = 1$

$\rightarrow f'(x) = 2x + 1$

$f(x) = \int f'(x)\,dx = \int (2x+1)\,dx = 2 \cdot \frac{x^2}{2} + x + C_2 = x^2 + x + C_2$

$f(2) = (2)^2 + (2) + C_2 = 4 + \cancel{2} + C_2 = 6 + C_2 = 10$

$\rightarrow C_2 = 4$

So $f(x) = x^2 + x + 4$

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3) A ball is thrown upward with initial velocity $64 \frac{ft}{s}$ from an initial height of 80 ft.

$s'(0) = 64$ $\qquad$ $s(0) = 80$

a) Find the position function $s(t)$ giving height as function of time

b) When does the ball hit the ground?

Given: $s''(t) = -32 \frac{ft}{s^2}$ is accel. due to gravity

$s'(t) = \int s''(t)\,dt = \int -32\,dt = -32t + C$

$s'(0) = -32(0) + C = 64 \rightarrow C_1 = 64$

$s(t) = \int s'(t)\,dt = \int (-32t + 64)\,dt$

$= -32 \cdot \frac{t^2}{2} + 64 \cdot t + C_2 = -16t^2 + 64t + C_2$

$s(0) = -16(0)^2 + 64(0) + C_2 = 80 \rightarrow C_2 = 80$

a) $\Longrightarrow$ $s(t) = -16t^2 + 64t + 80$

b) $-16t^2 + 64t + 80 = 0$

$t^2 - 4t - 5 = 0$

$(t+1)(t-5) = 0$

$t = -1$ or $t = 5$

Since $t > 0$, the ball hits the ground at $t = 5$ seconds

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Area

Def The sum of n terms $a_1, a_2, \ldots, a_n$ is written in sigma notation as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

where i is called the index of summation, a_i is the i^{th} term, and the upper and lower bounds of summation are n and 1.

Ex $\sum_{k=1}^{4} k = 1+2+3+4$

$$\sum_{i=2}^{4}\left(\frac{1}{\sqrt{i}}-3\right)^2 = \left(\frac{1}{\sqrt{2}}-3\right)^2 + \left(\frac{1}{\sqrt{3}}-3\right)^2 + \left(\frac{1}{\sqrt{4}}-3\right)^2$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x$$

$$\sum_{j=1}^{5} 2 = 2+2+2+2+2$$

Thm Properties of Summation

1) $\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$

2) $\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

3) $\sum_{i=1}^{n} c = cn$, c constant

4) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

5) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

6) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Ex Use sigma notation to rewrite the sum.

a) $\frac{9}{1+1} + \frac{9}{1+2} + \cdots + \frac{9}{1+14} = \sum_{k=1}^{14} \frac{9}{1+k}$

b) $\left(\left(\frac{2}{n}\right)^3 - \frac{2}{n}\right)\left(\frac{2}{n}\right) + \cdots + \left(\left(\frac{2n}{n}\right)^3 - \frac{2n}{n}\right)\left(\frac{2}{n}\right) = \sum_{i=1}^{n}\left(\left(\frac{2i}{n}\right)^3 - \frac{2i}{n}\right)\left(\frac{2}{n}\right)$