Math 2413
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Ex. Determine the open intervals on which the graph of $f(x)$ is concave up or concave down. Find inflection points and relative extrema.

) $f(x) = \dfrac{x^2+1}{x^2-4}$ $\qquad f'(x) = \dfrac{(x^2+1)'(x^2-4) - (x^2+1)(x^2-4)'}{(x^2-4)^2} = \dfrac{2x(x^2-4) - (x^2+1)(2x)}{(x^2-4)^2}$

$$f'(x) = \frac{-10x}{(x^2-4)^2}$$

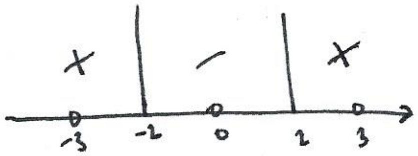
$$f''(x) = -\frac{(10x)'(x^2-4)^2 - 10x\left[(x^2-4)^2\right]'}{(x^2-4)^4} = -\frac{10(x^2-4)^2 - 10x \cdot 2(x^2-4)\cdot 2x}{(x^2-4)^4}$$

$$f''(x) = \frac{10(3x^2+4)}{(x^2-4)^3}$$

$f''(x) = 0$ implies $10(3x^2+4) = 0$
$3x^2+4 = 0$
$x^2 = \frac{-4}{3}$
no real solution

$f''(x)$ DNE implies $(x^2-4)^3 = 0$
$x^2-4 = 0$
$(x-2)(x+2) = 0$
$x = \pm 2$
$f(\pm 2)$ are not defined

f'' :



$f''(-3) = \dfrac{10\cdot(3\cdot 9+4)}{(9-4)^3} > 0$

$f''(0) = \dfrac{10\cdot 4}{(-4)^3} < 0$

$f''(3) = \dfrac{10\cdot(3\cdot 9+4)}{(9-4)^3} > 0$

* no inflection points

* f ∪ on $(-\infty,-2)\cup(2,\infty)$
f ∩ on $(-2,2)$

* $f'(x) = 0$ implies $10x = 0$
$x = 0$

Since $f''(0) < 0$, f has local max @ $x = 0$

no local min

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2) $g(x) = \sin\left(\frac{x}{2}\right),\ x \in [0, 4\pi]$

$g'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right)$

$g''(x) = -\frac{1}{4}\sin\left(\frac{x}{2}\right)$

all three functions defined and continuous on $[0, 4\pi]$

$g'(x) = 0$ implies $\frac{x}{2} = (2k+1)\frac{\pi}{2}$

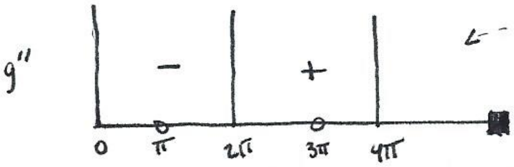
$x = (2k+1)\pi \rightarrow k = 0, 1$

$\underline{x = \pi, 3\pi}$

$g''(x) = 0$ implies $\frac{x}{2} = k\pi$

$x = 2k\pi \rightarrow k = 0, 1, 2$

$\underline{x = 0, 2\pi, 4\pi}$



$g''(\pi) = -\frac{1}{4}\sin\left(\frac{\pi}{2}\right) = -\frac{1}{4}$

$g''(3\pi) = -\frac{1}{4}\sin\left(\frac{3\pi}{2}\right) = \frac{1}{4}$

g $\cup$ on $(2\pi, 4\pi)$

g $\cap$ on $(0, 2\pi)$

Since $g''(\pi) < 0$, g has local max at $x = \pi$.

Since $g''(3\pi) > 0$, g has local min at $x = 3\pi$

g (x) has inflection point at $x = 2\pi$

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3) $f(x) = -3x^5 + 5x^3$

$f'(x) = -15x^4 + 15x^2$

$f''(x) = -60x^3 + 30x$

all are continuous on $\mathbb{R}$

$f''(x) = 0$ implies $-60x^3 + 30x = 0$

$30x(1 - 2(x^2)) = 0$

$\rightarrow \underline{x = 0}$

also $1 - 2(x^2) = 0$

$\rightarrow x = \pm\frac{1}{\sqrt{2}} =$ [illegible] $= \underline{\pm\frac{1}{\sqrt{2}}}$

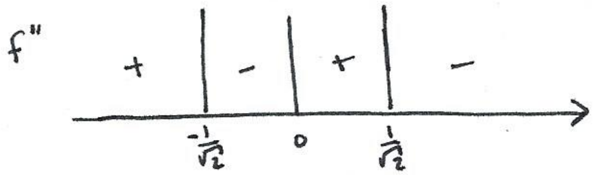
$f'(x) = 0$ implies $-15x^4 + 15x^2 = 0$

$15x^2(1 - x^2) = 0$

$\rightarrow \underline{x = 0}$

also $1 - x^2 = 0$

$\rightarrow \underline{x = \pm 1}$



$f''(-0.1) = -30(-0.1)(2(-0.1)^2 - 1) < 0$

$f''(0.1) = -30(0.1)(2(0.1)^2 - 1) > 0$

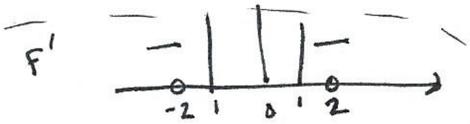
$f''(-1) = -30(-1)(2(-1)^2 - 1) > 0 \rightarrow$ local min at $x = -1$

$f''(1) = -30(1)(2(1)^2 - 1) < 0 \rightarrow$ local max at $x = 1$

$f(x)$ $\cup$ on $(-\infty, -\frac{1}{\sqrt{2}}) \cup (0, \frac{1}{\sqrt{2}})$

$f(x)$ $\cap$ on $(\frac{-1}{\sqrt{2}}, 0) \cup (\frac{1}{\sqrt{2}}, \infty)$

Since $f''(0) = 0$, 2nd Deriv Test fails - Use 1st Deriv Test



$f'(-2) =$

* ignore - mistake in class