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Differentials

$$\Delta y = f(x+\Delta x) - f(x) \approx dy$$
$$f(x+\Delta x) \approx f(x) + dy$$
$$f(x+\Delta x) \approx f(x) + f'(x)dx$$

Ex. Approximate $\sqrt{16.5}$ $\longmapsto$ $f(x) = \sqrt{x}$, $x = 16$, $\Delta x = 0.5 = \frac{1}{2}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$\sqrt{16.5} = \underbrace{\sqrt{16}}_{f(x)} + \underbrace{\frac{1}{2\sqrt{16}}}_{f'(x)}\underbrace{\frac{1}{2}}_{\Delta x} = 4 + \frac{1}{2\cdot 2\cdot 4} = 4 + \frac{1}{16} = 4.0625$$

INTEGRATION

Def A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ $\forall x \in I$.

Ex Find some antiderivatives of $f(x) = 2$.

$$\frac{d}{dx}(2x) = 2 \qquad \frac{d}{dx}(2x-1) = 2 \qquad \frac{d}{dx}(2x-c) = 2$$

Thm If F is an antiderivative of f on I, then the most general antiderivative of f on I is $F(x) + C$.

Def. 1) The constant C above is called the constant of integration.

2) An equation that involves the derivatives of a function is called a differential equation.

3) $G(x) = F(x) + C$ is the general solution of differential equation $G''(x) =$

4) When solving differntial equation of the form $\frac{dy}{dx} = f(x)$, it is convenient to write the equation in the equivalent form $dy = f(x)dx$.

5) The operation of finding all solutions to this eq. is called antidifferentiation or integration, and the symbol denoting this is $\int f(x)dx$, where $\int dx$ must always

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$$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$$

Ex. Find the indefinite integral.

1) $\int\left[5x^2+\frac{3}{x}-\frac{1}{x^{2/3}}\right]dx = \int 5x^2\,dx + \int \frac{3}{x}\,dx - \int x^{-2/3}\,dx$

$= 5\int x^2\,dx + 3\int \frac{1}{x}\,dx \quad \int x^{-2/3}\,dx$

$= 5\left(\frac{x^{2+1}}{2+1}+C_1\right)+3\left(\ln|x|+C_2\right)-\left(\frac{x^{-2/3+1}}{-2/3+1}+C_3\right)$

$= \frac{5}{3}x^3+3\ln|x|+3x^{1/3}+5C_1+3C_2-C_3$, let $C=5C_1+3C_2-C_1$

$= \frac{5}{3}x^3+3\ln|x|+\frac{3}{5}x^{5/3}+C$

Note:

$$\int x^n\,dx = \begin{cases}\frac{x^{n+1}}{n+1}+C, & n\neq -1\\ \ln|x|+C, & n=-1\end{cases}$$

2) $\int\frac{x+1}{\sqrt{x}}\,dx = \int\frac{x}{\sqrt{x}}\,dx+\int\frac{1}{\sqrt{x}}\,dx = \int x^{1/2}\,dx+\int x^{-1/2}\,dx = \frac{x^{3/2}}{3/2}+\frac{x^{1/2}}{1/2}+C = \frac{2}{3}x^{3/2}+2x^{1/2}+C$

* $\int[f(x)\pm g(x)]\,dx = \int f(x)\,dx + \int g(x)$, but $\int f(x)g(x)\,dx \neq \int f(x)\,dx\int g(x)\,dx$ and

$$\int\frac{f(x)}{g(x)}\,dx \neq \frac{\int f(x)\,dx}{\int g(x)\,dx}$$

3) $\int\frac{\sin(x)}{\cos^2(x)}\,dx = \int\frac{\sin(x)}{\cos(x)}\frac{1}{\cos(x)}\,dx = \int\tan(x)\sec(x)\,dx = \sec(x)+C$

4) $\int\left[(t^2+1)^2+t^{1/3}(t-4)\right]dt = \int\left[t^4+2t^2+1+t^{4/3}-4t^{1/3}\right]dt = \frac{t^{4+1}}{4+1}+2\cdot\frac{t^{2+1}}{2+1}+\frac{t^{0+1}}{0+1}+\frac{t^{4/3+1}}{4/3+1}-4\cdot\frac{t^{1/3+1}}{1/3+1}+C$

$= \frac{1}{5}t^5+\frac{2}{3}t^3+t+\frac{3}{7}t^{7/3}-4\cdot\frac{3}{4}t^{4/3}+C$

$= \frac{1}{5}t^5+\frac{2}{3}t^3+t+\frac{3}{7}t^{7/3}-3t^{4/3}+C$