Math 2413
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a) $f(x) = (x^2-4)^{\frac{2}{3}}$

① $f'(x) = \dfrac{4x}{3(x^2-4)^{1/3}}$

$f'(x) \Rightarrow x = 0$

$f'(x)$ DNE $\Rightarrow x^2-4=0$

$(x-2)(x+2)=0$

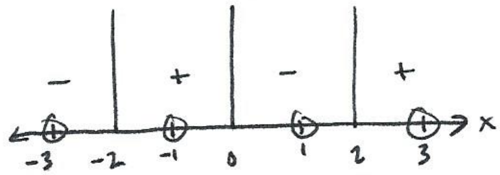
$x = -2, 2$

$f(x)$ defined on $(-\infty, \infty)$,

so critical numbers are

$x = -2, 0, 2$

②



$f'(-3) = \dfrac{4(-3)}{3((-3)^2-4)^{1/3}} < 0$

$f'(-1) = \dfrac{4(-1)}{3((-1)^2-4)^{1/3}} > 0$

$f'(1) = \dfrac{4(1)}{3((1)^2-4)^{1/3}} < 0$

$f'(3) = \dfrac{4(3)}{3((3)^2-4)^{1/3}} > 0$

③ f increasing on

$(-2,0) \cup (2,\infty)$

f decreasing on

$(-\infty,-2) \cup (0,2)$

f has min at $x = \pm 2$

max at $x = 0$

b) $f(x) = \dfrac{x^4+1}{x^2} = x^2 + \dfrac{1}{x^2}$

$f'(x) = 2x - \dfrac{2}{x^3} = \dfrac{2(x^4-1)}{x^3}$

$f'(x) = 0 \Rightarrow x^4 - 1 = 0$

$(x^2-1)(x^2+1) = 0$

$x^2 - 1 = 0$ $\quad\quad$ $x^2+1=0$

$(x-1)(x+1) = 0$ $\quad\quad$ no real solution

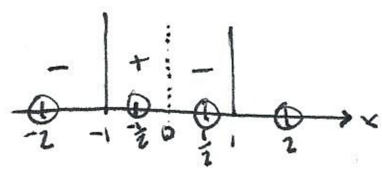
$x = \pm 1$

$f'(x)$ DNE $\Rightarrow x = 0$

Since f defined for all non zero reals,

the critical numbers are $x=-1$

$x = 1$



$f'(-2) = 2(-2) - \dfrac{2}{(-2)^3} = -4 + \dfrac{2}{8} < 0$

$f(-\tfrac{1}{2}) = 2(-\tfrac{1}{2}) - \dfrac{2}{(-\frac{1}{2})^3} = -1 + 2\cdot 8 > 0$

$f(\tfrac{1}{2}) = 1 - 2\cdot 8 < 0$

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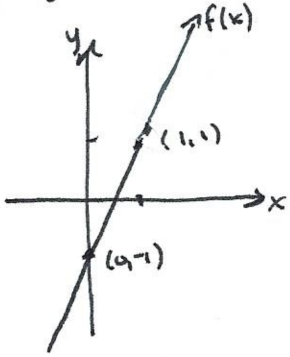
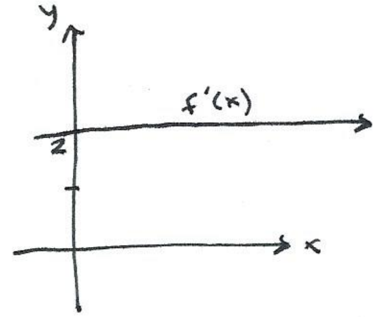
a) graph $f'(x)$



$m = \frac{\Delta y}{\Delta x}$

$= \frac{1-(-1)}{1-0}$

$= 2$

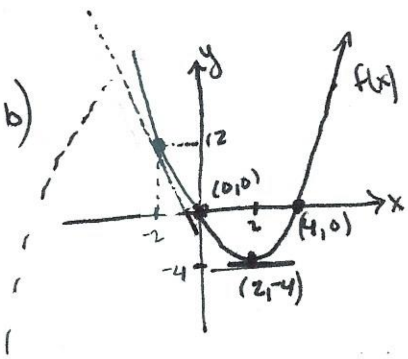


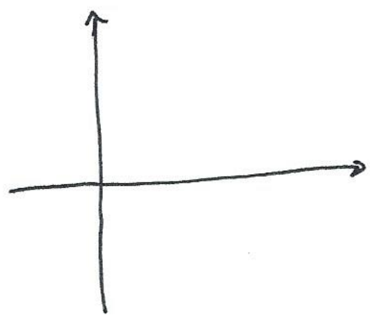
$y-(-1) = 2(x-0)$

$y = 2x - 1$

$y' = 2$

b)





x	0	2	4
f(x)	0	-4	0

x	-2	2	6
f'(x)	-8	0	6

$x = -2,\ y = 12$

$x = -\frac{1}{2},\ y = 0$

$m = \frac{12-0}{-2-\frac{1}{2}} = \frac{12}{-2+\frac{1}{2}} = -8$

$x = 4,\ y = 0$

$x = 6,\ y = 12$

$m = \frac{12-0}{6-4} = 6$

$f(x) = x(x-4)$

$f'(x) = 2x - 4$