Differentials

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Def. Let f be differentiable at c. The <u>linear approximation</u> of f at c is or <u>tangent line approximation</u>

$$y = f(c) + f'(c)(x-c)$$

Ex. Find the tangent line approximation of $f(x) = \sqrt{x}$ at $x=16$. use this to approximate $\sqrt{16.5}$.

$f(x) = x^{1/2}$
$f'(x) = \frac{1}{2}x^{-1/2}$
$f'(16) = \frac{1}{2}4^{-1} = \frac{1}{8}$
$f(16) = 4$

$y = f(16) + f'(16)(x-16)$
$= 4 + \frac{1}{8}(x-16)$
$= \frac{1}{8}(x) + 2$

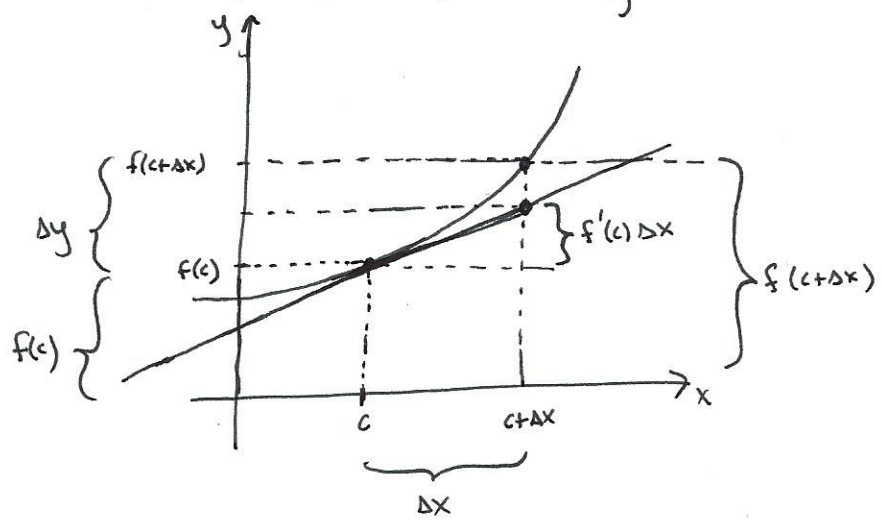
$\sqrt{x} \approx \frac{x}{8} + 2$ near $x=16$

$\sqrt{16.5} \approx \frac{16.5}{8} + 2 = \frac{16}{8} + \frac{\frac{1}{2}}{8} + 2 = 4 + \frac{1}{16} = 4.0625$

Def In $y = f(c) + f'(c)(x-c)$, $(x-c)$ is called the <u>change in x</u> and is denoted Δx.
when Δx is small, Δy can be approximated as

$$\Delta y = f(c+\Delta x) - f(c) \approx f'(c)\,\Delta x$$

Def. Let $y = f(x)$ be differetiable on an open interfal containing x. The <u>differential of x</u> denoted by dx is any nonzero real number. The <u>differential of y</u> is $dy = f'(x)\,dx$



Ex. Let $y = \sqrt{x}$. Find dy when $x=16$ and $dx = 0.5$. Compare with Δy for $x=16$ and $\Delta x = 0.5$

$dy = f'(x)\,dx = \frac{1}{2\sqrt{16}}\left(\frac{1}{2}\right) = \frac{1}{2\cdot 4}\frac{1}{2} = \frac{1}{16}$
≈ 0.0625

$\Delta y = \sqrt{16.5} - \sqrt{16} \approx 4.0620 - 4 = 0.0620$

Note: in many applications, dy is used to approximate Δy, such as in error propagation for physical measuring devices.

$$\underbrace{f(x + \overset{\text{measurement error}}{\Delta x})}_{\text{exact value}} - \overbrace{f(x)}^{\text{measured value}} = \overbrace{\Delta y}^{\text{error propagated}}$$

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Ex. The measured radius of a ball bearing is 0.7 inch, correct to within 0.01 inch. Estimate the propagated error in the Volume V of the ball bearing. Is the propagated error small or large?

$V = \frac{4}{3}\pi r^3$, $r = 0.7$, $-0.01 \le \Delta r \le 0.01$

$\Delta V = V(0.7+\Delta r) - V(0.7) \approx dV = 4\pi r^2 dr = 4\pi(0.7)^2(\pm 0.01) \approx \pm 0.06158$

The volume has a propagated error of about 0.06158 cubic inch

Relative error $= \frac{dV}{V} = \frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} = \frac{3dr}{r} = \frac{3}{0.7}(\pm 0.01) \approx \pm 0.0429$.

The percent error in volume measurement is approximately 4.29%

Ex. Find differential of $y = \sin(3x)$

$dy = \cos(3x)(3x)' dx$

$= 3\cos(3x)dx$