Math 2413
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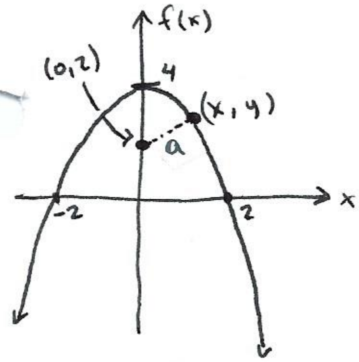
Ex. Which points on the graph of $f(x) = 4 - x^2$ are closest to the point $(0,2)$?



minimize: $a = \sqrt{(x-0)^2 + (y-2)^2}$

$= \sqrt{x^2 + (4-x^2-2)^2} = \sqrt{x^2 + (2-x^2)^2}$

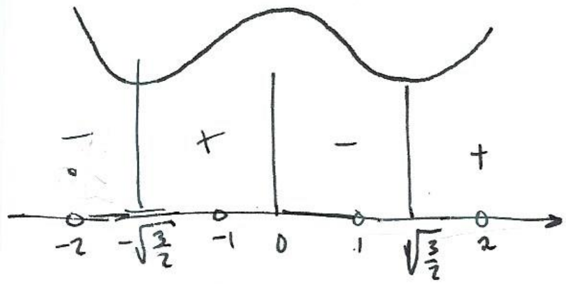
$= \sqrt{x^2 + 4 + x^4 - 4x^2} = \sqrt{x^4 - 3x^2 + 4}$

$\frac{da}{dx} = 0 = \frac{4x^3 - 6x}{2\sqrt{x^4 - 3x^2 + 4}} \longmapsto 4x^3 - 6x = 0$

$2x(2x^2 - 3) = 0$

$\underline{x = 0}$ or $2x^2 - 3 = 0$

$x = \pm\sqrt{\frac{3}{2}}$



$\frac{da}{dx}$ DNE $\rightarrow x^4 - 3x^2 + 4 = 0$

Let $t = x^2$. Then $t^2 - 3t + 4 = 0$

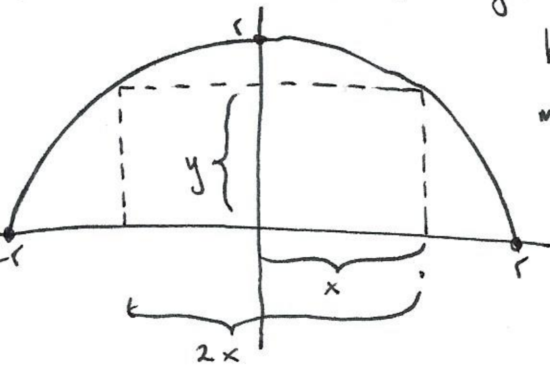
$t = \frac{1}{2}\left[3 \pm (9-16)^{1/2}\right] \notin \mathbb{R}$

$\longrightarrow$ no such x exists

$\Longrightarrow$ when $x = \pm\sqrt{\frac{3}{2}}$, the dist is min

$\Longrightarrow \left(-\sqrt{\frac{3}{2}}, 4 - \frac{3}{2}\right)$ and $\left(\sqrt{\frac{3}{2}}, 4 - \frac{3}{2}\right)$ are the points of f which are closest to the point $(0,2)$.

Ex. Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.



known: $x^2 + y^2 = r^2$

maximize: $A = 2xy = 2x(r^2 - x^2)^{1/2} = 2(r^2x^2 - x^4)^{1/2}$

$\rightarrow$ minimize $r^2x^2 - x^4$

$0 = \frac{d}{dx}\left[r^2x^2 - x^4\right] = 2r^2x - 4x^3 = 0$

$x(2r^2 - 4x^2) = 0$

$\underline{x = 0}$ or $2r^2 - 4x^2 = 0$

$\frac{1}{2}r^2 = x^2$

$x = \pm\frac{r}{\sqrt{2}}$

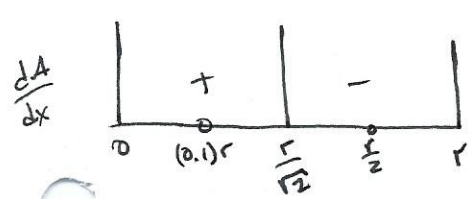
$\longrightarrow \underline{x = +\frac{r}{\sqrt{2}}}$

$\frac{dA}{dx} = \frac{2r^2x - 4x^3}{2(r^2x^2 - x^4)^{1/2}}$

$\frac{dA}{dx}$ DNE $\rightarrow r^2x^2 - x^4 = 0$

$x^2(r^2 - x^2) = 0$

$\underline{x = 0}$ or $\underline{x = \pm r}$



The largest area is r^2.

$A(0) = A(r) = 0$

$A\left(\frac{r}{\sqrt{2}}\right) = r^2$