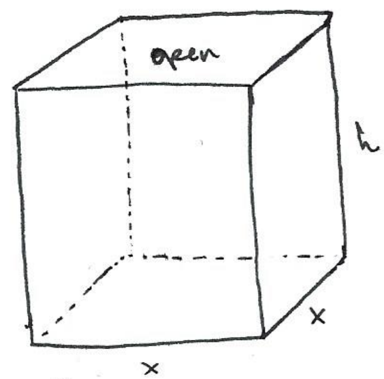
Optimization

1) Identify unknown quantities and known quantities
2) Write primary equation to be maximized or minimized
3) reduce primary equation to one involving only one variable
4) Determine domain of primary equation
5) Compute desired min/max by calculus techniques.

Ex. Manufacturer wants open box with square base and surface area of 108 sq. inches. What dimensions produce maximum volume?



Known: $SA = 108 = x^2 + 4xh \longrightarrow h = \frac{108-x^2}{4x}$

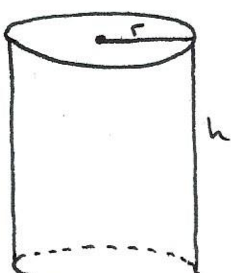
maximize: $V = x^2 h = \frac{1}{4}x^2 \cdot \frac{108-x^2}{x} = \frac{1}{4}(108x - x^3)$

$$\frac{dV}{dx} = 0 = \frac{108}{4} - \frac{3}{4}x^2 \rightarrow 3x^2 = 108$$
$$x^2 = 36$$
$$x = \pm 6 \mapsto \boxed{x=6}$$, since dimensions must be pos #

Since $x=6$, $h = \frac{108-6^2}{24}$

$= \frac{72}{24} = 3 \implies$ dim $= 6'' \times 6'' \times 3''$

Ex A cylindrical can is made to hold 1L of oil. Find dimensions to minimize the cost of the metal to make the can.



Known: $V = 1L = 1000\text{ cm}^3 = \pi r^2 h \mapsto h = \frac{1000}{\pi r^2}$

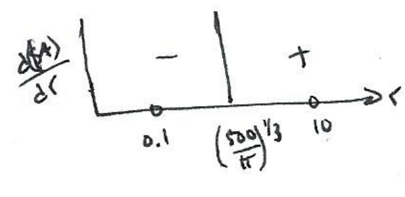
minimize: $SA = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \cdot \frac{1000}{\pi r^2} = 2\pi r^2 + \frac{2000}{r}$

$0 = \frac{d(SA)}{dr} = 4\pi r - \frac{2000}{r^2} \mapsto 4\pi r - \frac{2000}{r^2} = 0$

$$4\pi r^3 = 2000$$
$$r^3 = \frac{500}{\pi}$$
$$r = \left(\frac{500}{\pi}\right)^{1/3}$$
$$\longrightarrow h = \frac{1000}{\pi\left(\frac{500}{\pi}\right)^{2/3}}$$

So the radius should be
$r = \left(\frac{500}{\pi}\right)^{1/3}$ cm

and the height should be
$h = \frac{1000}{\pi\left(\frac{500}{\pi}\right)^{2/3}}$ cm



Ex A rectangular page is to contain 24 sq. in of print. The top and bottom margins should be 1.5 in and the left and right margins should be 1 in. What should be the dimensions of the page to use the least amount of paper?

Known: $(x-2)(y-3) = 24$

$$y = \frac{24}{x-2} + 3$$

Minimize: $A = xy = x\left(\frac{24}{x-2}+3\right) = \frac{24x}{x-2} + 3x$

$$\frac{dA}{dx} = 0 = \frac{24(x-2) - 24x}{(x-2)^2} + 3 = \frac{24(x-2-x)}{(x-2)^2} + 3$$

$$= \frac{-48}{(x-2)^2} + 3$$

$$\frac{48}{(x-2)^2} = 3$$

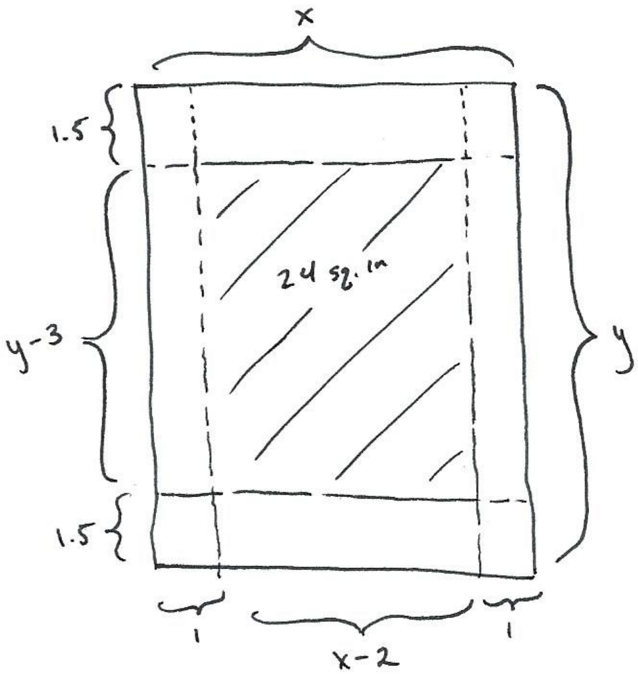
$$16 = (x-2)^2$$

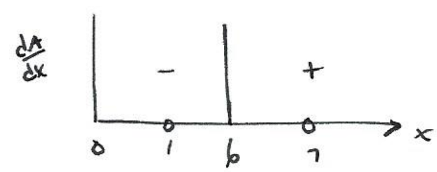
$\pm 4 = x-2 \longrightarrow x = 2 \pm 4$, but since $2-4<0$,

$x = 2+4 = 6$

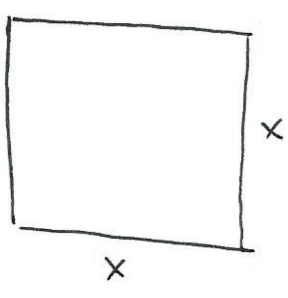
$\longrightarrow y = \frac{24}{6-2} + 3 = 6+3 = 9$

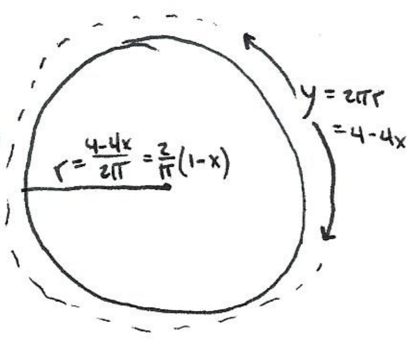
So the paper should measure 6" x 9"





Ex. 4 feet of wire will used to form a square and a circle. How much should be used for each to enclose the max total area?





Known: $4x + 2\pi r = 4$

Maximize: $A = x^2 + \pi r^2$

$$= x^2 + \pi\left(\frac{2}{\pi}(1-x)\right)^2$$

$$= x^2 + \frac{4}{\pi}(1-x)^2$$

$$\frac{dA}{dx} = 0 = 2x + \frac{8}{\pi}(1-x)(-1) = 2x - \frac{8}{\pi}(1-x)$$

$$\frac{8}{\pi}(1-x) = \frac{8}{\pi} - \frac{8x}{\pi} = 2x$$

$$\frac{8}{\pi} = \left(\frac{8}{\pi}+2\right)x$$

$$x = \frac{8/\pi}{\frac{8}{\pi}+2} = \frac{8}{8+2\pi} = \frac{4}{4+\pi}$$

$$\rightarrow 4x = \frac{16}{4+\pi}$$

$$4-4x = \frac{16+16\pi-16}{4+\pi} = \frac{16\pi}{4+\pi}$$