Increasing & Decreasing Functions and the First Derivative Test

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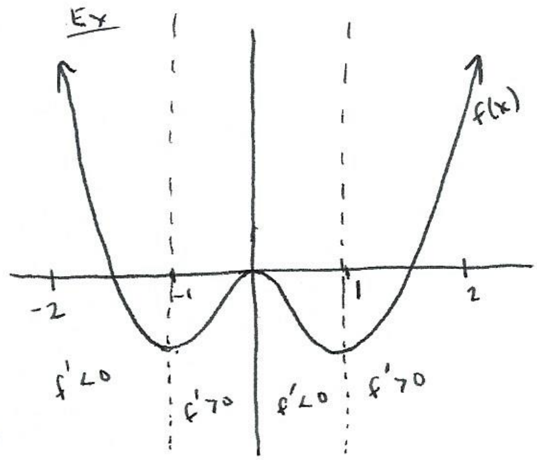
Def A function f is increasing on an interval if for any two numbers x_1 and x_2 on an interval $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

f is decreasing if $x_1 < x_2$ implies $f(x_1) > f(x_2)$

f is strictly monotonic on an interval if f is increasing on the entire interval or decreasing " "

Ex



$f > 0$ on $(-\infty, -\frac{3}{2}) \cup (\frac{3}{2}, \infty)$

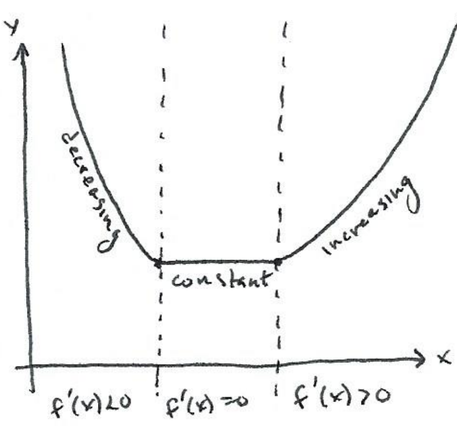
$f < 0$ on $(-\frac{3}{2}, 0) \cup (0, \frac{3}{2})$

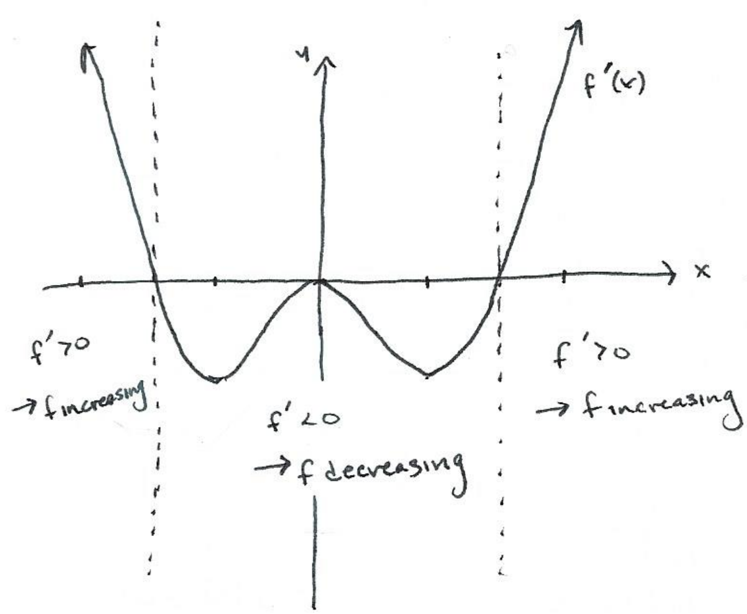
f increasing on $(-\frac{3}{2}, 0) \cup (\frac{3}{2}, \infty)$

f decreasing on $(-\infty, -\frac{3}{2}) \cup (0, \frac{3}{2})$

Thm. Let f be continuous on $[a,b]$ and differentiable on (a,b). Then:

1) if $f'(x) > 0$ for all $x \in (a,b)$, then f is increasing on $[a,b]$.
2) if $f'(x) < 0$ for all $x \in (a,b)$, then f is decreasing on $[a,b]$.
3) if $f'(x) = 0$ for all $x \in (a,b)$, then f is constant on $[a,b]$.





<u>Note</u>: Let f continuous on (a,b). To find the open intervals on which f is increasing or decreasing,

1) locate critical numbers of f in (a,b), and use these to determine test intervals

2) Determine the sign of $f'(x)$ at one test point in each interval

3) Use the test for increasing or decreasing functions to determine whether f is increasing or decreasing on each interval.

<u>Ex</u>. $f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

$f'(x) = 0 \Rightarrow 3x^2 - 3x = 0$

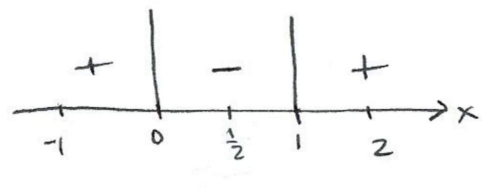
$3x(x-1) = 0$

$\rightarrow x = 0, 1$

$f'(x)$ DNE $\Rightarrow$ no such x

f is a polynomial, so f is defined at $\underline{x=0}$ and $\underline{x=1}$.

These are the critical numbers



$f'(-1) = 3(-1)^2 - 3(-1) = 3 + 3 = \underline{6 > 0}$

$f'(\frac{1}{2}) = 3(\frac{1}{2})^2 - 3(\frac{1}{2}) = \frac{3}{4} - \frac{3}{2} = \underline{-\frac{3}{4} < 0}$

$f'(2) = 3(2)^2 - 3(2) = 12 - 6 = \underline{6 > 0}$

$\Longrightarrow$ f is increasing on $(-\infty, 0) \cup (1, \infty)$

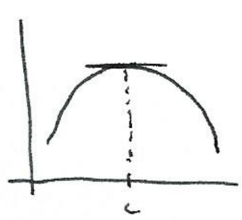
f is decreasing on $(0, 1)$

<u>Thm</u> <u>First Derivative Test</u>

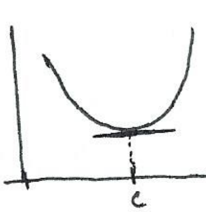
Let c be a critical number of a function f that is continuous on an open interval containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows:

1) If f' changes from negative to positive at c, then f has relative minimum at $(c, f(c))$.
2) " " positive to negative " " maximum @ $(c, f(c))$

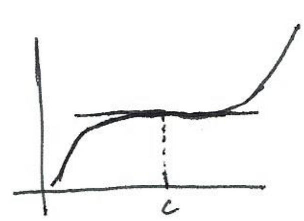
3) If f' does not change sing at c, then f has neither relative max nor relative min at $(c, f(c))$



local/relative max



local/relative min



no extremum here

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Ex. Find the local (relative) extrema

a) $f(x) = (x^2-4)^{2/3}$

$f'(x) = \frac{2}{3}(x^2-4)^{-1/3}(2x) = \frac{4}{3}x(x^2-4)^{-1/3}$

$f'(x) = 0 \Rightarrow x = 0$

$f'(x)$ DNE $\Rightarrow x^2-4 = 0$

$(x-2)(x+2) = 0$

$x = -2, 2$