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b) $g(x) = 2x^{5/3} - 5x^{4/3}$, Domain $= \mathbb{R}$, Range =

x-intercept: $g(0) = 0 - 0 = 0 = y$

y-intercept: $0 = 2x^{5/3} - 5x^{4/3}$

$= x^{4/3}(2x^{1/3} - 5)$

$x^{4/3} = 0$ $\quad$ $2x^{1/3} - 5 = 0$

$x = 0$ $\quad$ $x^{1/3} = \frac{5}{2}$

$x = \left(\frac{5}{2}\right)^3 = \frac{125}{8} = 15.625 = x$

Horizontal Asymptote:

$\lim_{x \to \infty} 2x^{5/3} - 5x^{4/3} = \infty$

$\lim_{x \to \infty} 2x^{5/3} - 5x^{4/3} = -\infty - \infty = -\infty$

no horiz asymptotes

critical points:

$g'(x) = \frac{10}{3}x^{2/3} - \frac{20}{3}x^{1/3} = \frac{10}{3}x^{1/3}(x^{1/3} - 2)$

$g'(x) = 0 \rightarrow \frac{10}{3}x^{1/3}(x^{1/3} - 2) = 0$

$\frac{10}{3}x^{1/3} = 0$ $\quad$ $x^{1/3} - 2 = 0$

$x = 0$ $\quad$ $x^{1/3} = 2$

$x = 8$

$g'(x)$ DNE $\rightarrow$ no such x

$g(0)$ and $g(8)$ are defined, so

crit pts are $x = 0, 8$

$g'(0) = 0$

inflection points:

$g''(x) = \frac{20}{9}x^{-1/3} - \frac{20}{9}x^{-2/3} = \frac{20}{9}x^{-2/3}(x^{1/3} - 1)$

$g''(x) = 0 \rightarrow \frac{20}{9}x^{-2/3}(x^{1/3} - 1) = 0$

$\frac{20}{9}x^{-2/3} = 0$ $\quad$ $x^{1/3} - 1 = 0$

$x = 0$ $\quad$ $x = 1$

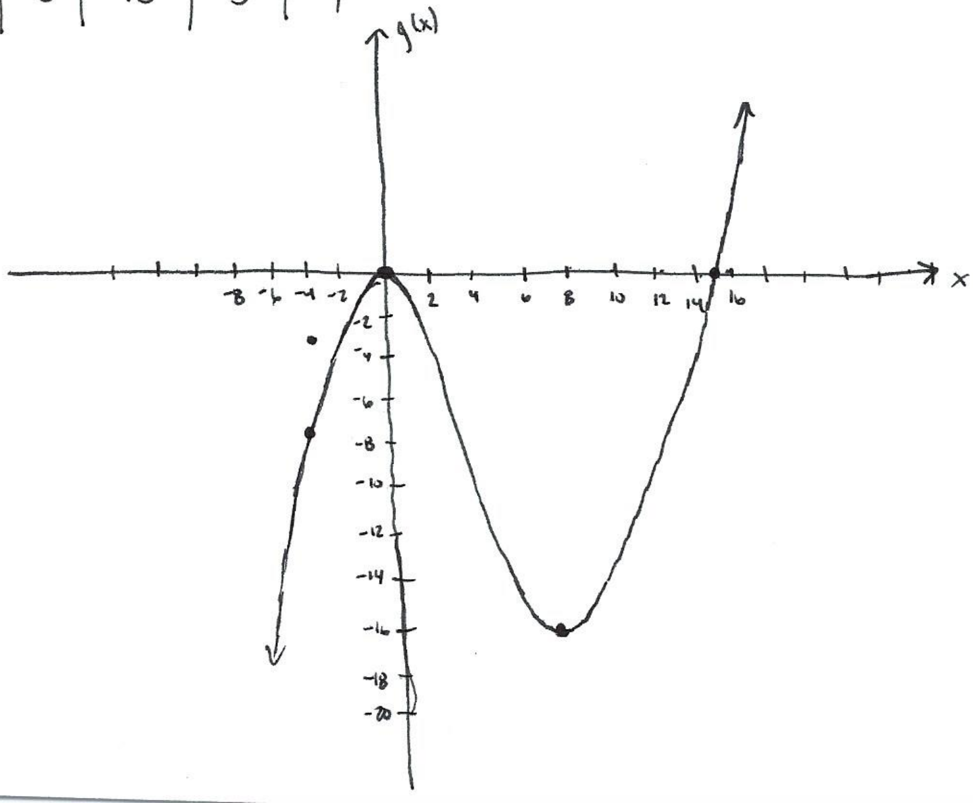
$g''(x)$ DNE $\rightarrow$ no such x

$g(0)$ and $g(1)$ are defined, so

possible inflection points are $x = 0, 1$

	$x<0$	$0<x<1$	$1<x<8$	$x>8$
test x	-1	$\frac{1}{2}$	2	9
f	↑ ∩	↓ ∩	↓ ∪	↑ ∪
f′	+	−	−	+
f″	−	−	+	+

x	0	$\frac{125}{8}$	8	1	−1
g(x)	0	0	−16	−3	−7



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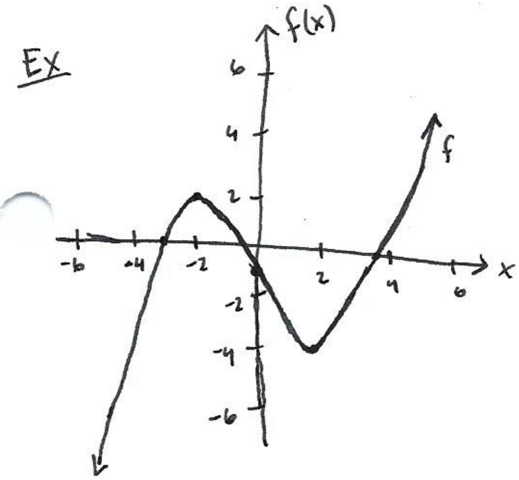
Ex



a) Determine critical points and inflection points

~~crit~~ inflection points: $x = 0 \Rightarrow$ <u>$(0,-1)$</u>

<u>crit points</u>: $f' = 0 \rightarrow x = \pm 2$

$\Rightarrow$ <u>$(-2,2)$ and $(2,-4)$</u>

b) Where is f increasing? Decreasing?
" " concave up? concave down?

<u>f inc</u> on $(-\infty,-2) \cup (2,\infty)$

<u>f dec</u> on $(-2,2)$

f <u>concave up</u> on $(0,\infty)$

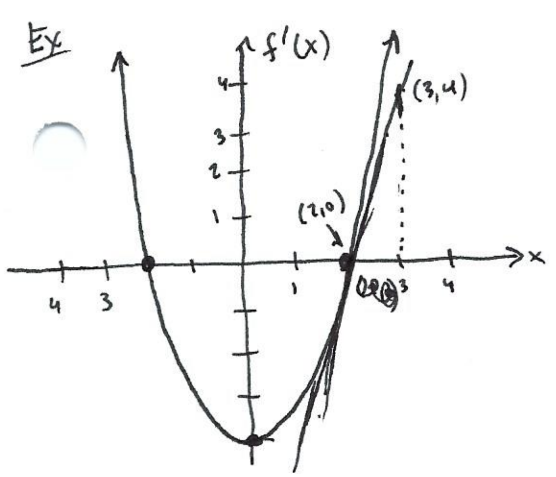
f <u>concave down</u> on $(-\infty,0)$

c) <u>relative extrema:</u>

local max at $(-2,2)$

local min at $(2,-4)$

Ex



a) Approximate slope of f at $x = 2$

~~$\frac{4-0}{3-2} = 4$~~ $f'(2) = 0$

b) Determine crit points and infl points

<u>crit #:</u> $f'(x) = 0$ (f' defined everywhere)

$\rightarrow$ <u>$x = \pm 2$</u> are the crit #s

<u>infl points</u>: x such that f(x) changes concavity

$\rightarrow$ <u>$x = 0$</u> is the only infl point

c) <u>f is ↑</u> if $f' > 0$

$\rightarrow (-\infty,-2) \cup (2,\infty)$

<u>f is ↓</u> if $f' < 0$

$\rightarrow (-2,2)$

<u>f is ∪</u> if $f'' > 0$ (means f' ↑)

$\rightarrow (0,\infty)$

f is ∩ if $f'' < 0$ (means f' ↓)

$\rightarrow (-\infty,0)$

d) f has a <u>local min</u> at x if $f'(x) = 0$ and f' changes from − to +.

$\rightarrow$ <u>$x = 2$</u>

f has <u>local max</u> at x if $f'(x)$ and f' changes from + to −.

$\rightarrow$ <u>$x = -2$</u>