Ex. (cont)

g) $\lim_{x\to\infty} \dfrac{(x^2-1)^{1/2}}{2x-1} = \lim_{x\to\infty}\left[\dfrac{x^2-1}{(2x-1)^2}\right]^{1/2} = \lim_{x\to\infty}\left(\dfrac{x^2-1}{4x^2-4x+1}\right)^{1/2}$

$= \lim_{x\to\infty}\left(\dfrac{1-\frac{1}{x^2}}{4-\frac{4}{x}+\frac{1}{x^2}}\right)^{1/2} = \left(\frac{1}{4}\right)^{1/2} = \underline{\frac{1}{2}}$

h) $\lim_{x\to\infty}\dfrac{x-\cos(x)}{x} = \lim_{x\to\infty} 1 - \dfrac{\cos(x)}{x}$

$= \underline{1}$

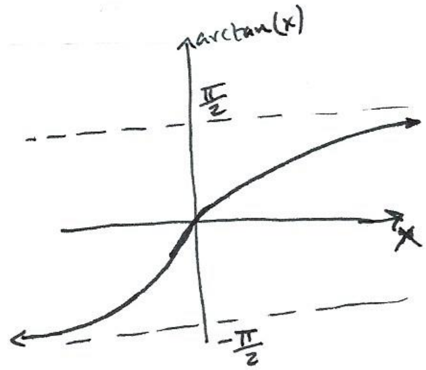
$*$ Since $-1 \le \cos(x) \le 1$

$-\frac{1}{x} \le \frac{\cos(x)}{x} \le \frac{1}{x}$

$\lim_{x\to\infty} -\frac{1}{x} = 0 = \lim_{x\to\infty}\frac{1}{x}$

i) $\lim_{x\to\infty}\left[\ln\left(\dfrac{x^2+1}{x^2}\right) - \arctan(x)\right] = \lim_{x\to\infty} \ln\left(1+\frac{1}{x^2}\right) - \arctan(x)$

$= \ln(1) - \frac{\pi}{2} = -\frac{\pi}{2}$



Ex Find the horizontal asymptotes of f(x)

$f(x) = \dfrac{1}{1+e^{-x}}$

$\lim_{x\to\infty}\dfrac{1}{1+e^{-x}} = \lim_{x\to\infty}\dfrac{1}{1+\frac{1}{e^x}} = 1$

$\lim_{x\to-\infty}\dfrac{1}{1+e^{-x}} = 0$

<u>$y=1$ and $y=0$ are two horizontal asymptotes</u>