Math 2413
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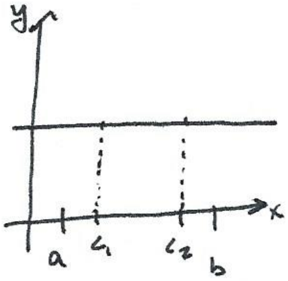
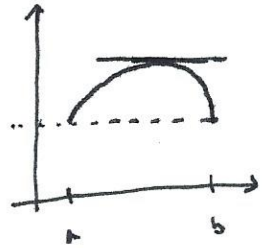
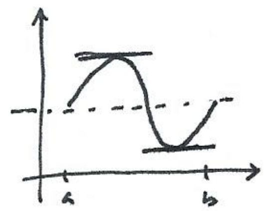
Pg 37

Rolle's Theorem - Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

If $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$.







Ex. Find the two x-intercepts of $f(x) = x^2 - 3x + 2$ and show that $f'(c) = 0$ at some point c between the two x-intercepts.

$f'(x) = 2x - 3$

$f'(c) = 2c - 3 = 0,$

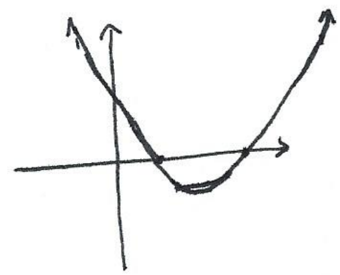
$2c = 3$

$c = \frac{3}{2}$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$



Since f is continuous on $[1, 2]$ and differentiable on $(1,2)$, and since $f(1) = f(2)$, Rolle's theorem guarantees the existence of some c in $(1,2)$ s.t. $f'(c) = 0$.

Ex. Rolls theorem cannot be applied to $f(x) = \left|\frac{1}{x}\right|$ on $[-1, 1]$ since f not continuous at $x = 0$

Ex. $f(x) = \sin(x)$ on $[0, 2\pi]$

$f(x)$ continuous on $[0, 2\pi]$

$f'(x) = \cos(x)$ differentiable on $(0, 2\pi)$

$f(0) = 0 = f(2\pi)$

By Rolle's Theorem, $\exists\, c \in (0, 2\pi)$ s.t. $f'(c) = 0$.

$\cos(c) = 0$

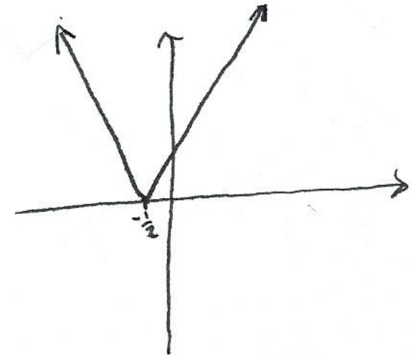
$c = \frac{\pi}{2}, \frac{3\pi}{2}$

(MVT)

<u>Mean Value Theorem</u> - If f is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), then there is at least one number c in (a,b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Ex. Determine whether MVT can be applied.

a) $f(x) = |2x+1|$ on $[-1,3]$



f continuous on $[-1,3]$, but $f'(-\frac{1}{2})$ does not exist,
so f not differentiable on $(-1,3)$.
MVT not applicable.

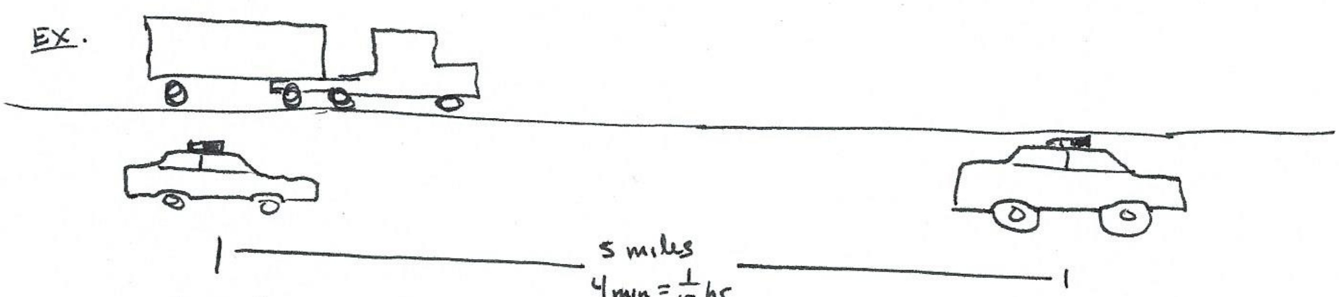
b) $f(x) = x\log_2 x$ on $[1,2]$

$f(x)$ is continuous on $[1,2]$ and differentiable on $(1,2)$.
Then by MVT, $\exists\, c \in (1,2)$ s.t. $f'(c) = \dfrac{f(2)-f(1)}{2-1} = \dfrac{2\cdot\log_2(2) - 1\cdot\log_2(1)}{1} = 2\cdot(1) - 0 = 2$

$$f'(x) = (x')\log_2(x) + x\left(\log_2 x\right)'$$
$$= \log_2 x + x\cdot\frac{1}{\ln(2)}\cdot\frac{1}{x}$$
$$= \log_2 x + \frac{1}{\ln(2)}$$

$$f'(c) = 2 = \log_2(c) + \frac{1}{\ln(2)}$$
$$\log_2(c) = 2 - \frac{1}{\ln(2)}$$
$$c = 2^{2-\frac{1}{\ln(2)}}$$

Ex.



$t=0$: truck speed = 55 mph

$t=\frac{1}{15}$: truck speed = 50 mph

Assume: distance function $s(t)$ is continuous on $[0,\frac{1}{15}]$ and $s'(t)$ exists on $(0,\frac{1}{15})$.

Then by MVT, $\exists\, c \in (0,\frac{1}{15})$ s.t. $|s'(c)| = \left|\dfrac{s(\frac{1}{15}) - s(0)}{\frac{1}{15} - 0}\right| = \left|\dfrac{50-55}{\frac{1}{15}}\right| = \left|\dfrac{-5}{\frac{1}{15}}\right| = 75$

recall speed $= |\text{velocity}|$
$$= |s'(t)|$$