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MATH 2312
Pre-Calculus II
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ex. $\sin(\cos^{-1}(1/2)) + \sin^{-1}(3/5)$

Find the exact value B.

$\alpha = \cos^{-1} 1/2$

$\cos\alpha = 1/2$

$0 \le \alpha \le \pi$

$B = \sin^{-1}(3/5)$

$\sin B = 3/5$

$-\pi/2 \le B \le \pi/2$

QI QIV

in this QI cos is (+).

$\cos^2 B + \sin^2 B = 1$

$\cos^2 B + (3/5)^2 = 1$

$\cos^2 (B) = 1 - \frac{9}{25}$

$= +\left(\frac{4}{5}\right)$

$\cos^2\alpha + \sin^2\alpha = 1$

$(1/2)^2 + \sin^2\alpha = 1$

$\sin^2\alpha = 3/4$

$\sin\alpha = +\frac{\sqrt{3}}{2}$

$\sin\alpha\cos B + \cos\alpha\sin B$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{4}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{5}\right)$.

$= \frac{4\sqrt{3}}{10} + \frac{3}{10}$

$= \frac{1}{10}(4\sqrt{3} + 3)$

$\sin(\sin^{-1}u + \cos^{-1}v)$

$\sin(\alpha + B) = \sin\alpha\cos B + \cos\alpha\sin B$.

$\sin^{-1}u = \alpha \qquad \cos^{-1}v = B$

$-1 \le u \le 1 \qquad -\pi/2 \le \alpha \le \pi/2$

$-1 \le v \le 1 \qquad 0 \le B \le \pi$

$\sin^{-1} u = \alpha$

$\sin\alpha = u$

$\cos^{-1} v = \beta$

$\cos\beta = v$

$\cos^2\alpha + \sin^2\alpha = 1$

$\cos^2\alpha + (u^2) = 1$

$\cos^2\alpha = 1 - u^2$

$\cos\alpha = \sqrt{1-u^2}$

$\cos\alpha = +\sqrt{1-u^2}$

$\sin\beta = \sqrt{1-v^2}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= uv + \sqrt{1+u^2}\,\sqrt{1-v^2}$

solve:

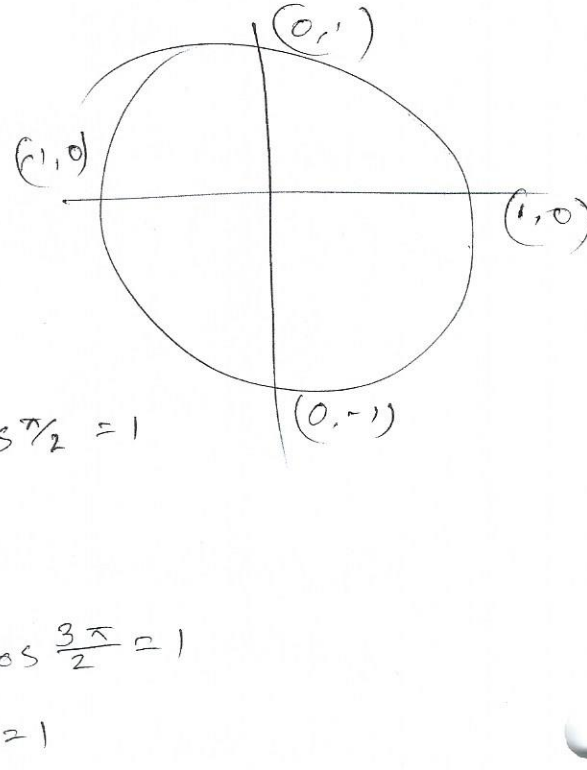
$(\sin\theta + \cos\theta)^2 = 1^2$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1$

$2\sin\theta\cos\theta = 0$

$\sin = 0$ or $\cos\theta = 0$

$\theta = 0, \pi$ $\quad \theta = \frac{\pi}{2}, \frac{3\pi}{2}$.



$\sin 0 + \cos 0 = 1$

$(0) + (1) = 1$

$\sin\pi + \cos\pi$

$0 + (-1) = 1$

$-1 \neq 1$

$\sin\pi/2 + \cos\pi/2 = 1$

$1 + 0 = 1$

$1 = 1$

$\sin\frac{3\pi}{2} + \cos\frac{3\pi}{2} = 1$

$-1 + 0 = 1$

$-1 \neq 1$

$0 \le \theta < 2\pi$

$\sin\theta + \cos\theta = 1$ divided by $\sqrt{2}$.

$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\phi \sin\theta + \sin\phi \cos\theta = \frac{1}{\sqrt{2}}$

$\cos\phi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\sin\phi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$,

$\frac{\pi}{4}, \frac{2\pi}{4}$ $\boxed{\phi = \frac{\pi}{4}}$

9.1 Law of sines

If a triangle is not a right triangle we call it oblique.

2 cases for oblique:

① acute triangle: all angles acute $< 90°$

② obtuse triangle: one angle obtuse $> 90°$

Theorem: Law of sines for a triangle sides

a, b, c angles A, B, C.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

4 cases of known information

(i) one side, two angles

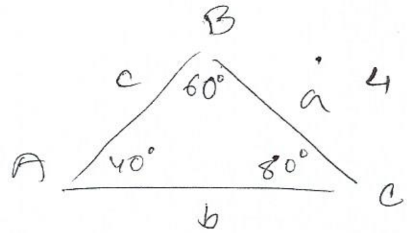
(ii) two sides, opposite angle

(iii) two sides, included angle

(iv) three side

ex. solve $\triangle$ SAA

$B = 60°$
$A = 40°$
$a = 4$



$\angle C = 80°$

$b = 5.39$

$c = 6.13$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 40}{4} = \frac{\sin 60}{b}$$

$$b = \frac{4 \sin 60}{\sin 40}$$

$$\approx 5.39$$

$$\frac{\sin 40°}{4} = \frac{\sin 80°}{c}$$

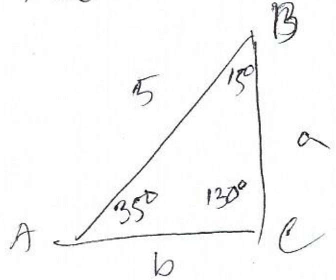
$$c = 4\frac{\sin 80°}{\sin 40°}$$

$$\approx 6.13$$

solve $\triangle$ ASA

$A = 35°$
$c = 5$
$B = 15°$

$C = 180 - (35+15)$
$C = 130°$



$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 35}{a} = \frac{\sin 130}{5}$$

$$a = \frac{5 \sin 35}{\sin 130}$$

$a = 3.74$.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 150°}{b} = \frac{\sin 130}{5}$$

$$b = \frac{5 \sin 15°}{\sin 130°}$$

$b \approx 1.69$