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MATH 2312
Pre-Calculus II
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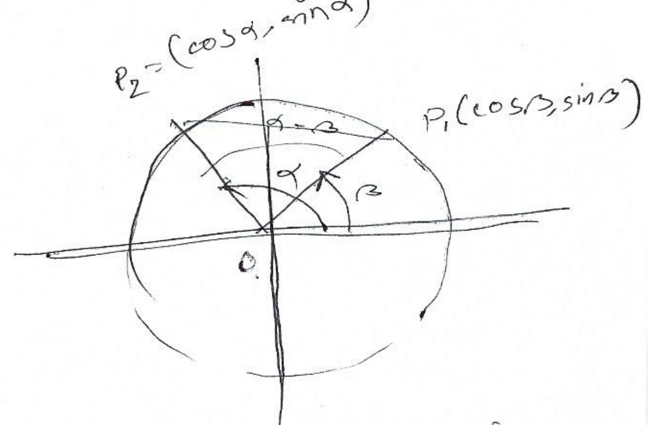
8.5 sum & difference Formulas

Theorem

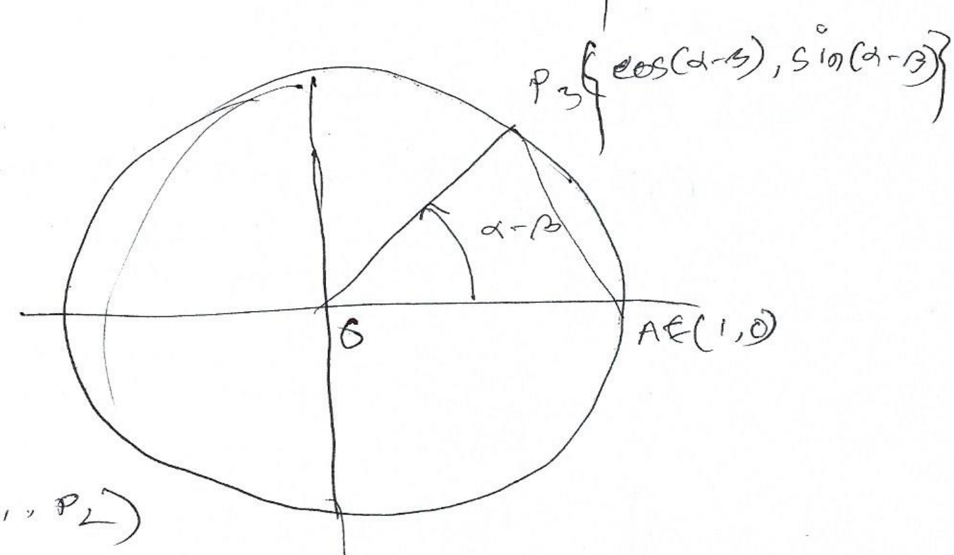
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

$0 < \beta < \alpha < 2\pi$



$\Delta OP_1P_2 \cong \Delta OAP_3$



$d(A, P_3) = d(P_1, P_2)$

$$\sqrt{(\cos(\alpha-\beta)-1)^2+(\sin(\alpha-\beta)-0)^2} = \sqrt{(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2}.$$

$$\Rightarrow \{\cos(\alpha-\beta)-1\}^2 + \sin^2(\alpha-\beta) = (\cos\alpha-\cos\beta)^2 + (\sin\alpha-\sin\beta)^2$$

$\Rightarrow \cos^2(\alpha-\beta) - 2\cos(\alpha-\beta) + 1 + \sin^2(\alpha-\beta) = \cos^2\alpha - 2\cos\alpha\cos\beta +$

$\cos^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta.$

$\Rightarrow \cancel{2} - 2\cos(\alpha-\beta)\} = \cancel{2} - 2\cos\alpha\cos\beta$

$- 2\sin\alpha\sin\beta$

$\cancel{\Rightarrow 2(1-\cos(\alpha-}$

$\Rightarrow \cancel{-}\cancel{2}\cos(\alpha-\beta) = \cancel{-}\cancel{2}(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$\cos 75^\circ$

$= \cos(45^\circ + 30^\circ)$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}.$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{(\sqrt{6}+\sqrt{2})}{4}$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\cdot\sin\theta$$

$$= 0\,\cos\theta + 1\,\sin\theta$$

$$= \sin\theta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\sin\frac{2\pi}{12} = \sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$$

$$= \sin\pi/3\,\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

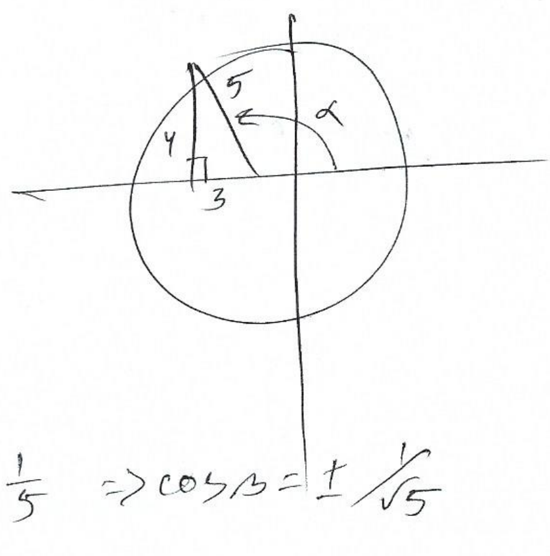
$$\sin 80^\circ\cos 20^\circ - \cos 80^\circ \sin 20^\circ$$

$$\sin(80^\circ - 20^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

ex. $\sin\alpha = 4/5 = y/r$ $\qquad \frac{\pi}{2} < \alpha < \pi$ $\qquad$ Q II

$\sin\beta = -2/\sqrt{5}$ $\qquad \pi < \beta < \frac{3\pi}{2}$ $\qquad$ Q III

$\cos\alpha = -3/5 = x/r$ $\qquad \cos\beta$



$\cos(\alpha+\beta)$

$\sin(\alpha+\beta)$

$\cos^2\beta + \sin^2\beta = 1$

$\cos^2\beta + \left(-2/\sqrt{5}\right)^2 = 1$

$\cos^2\beta + 4/5 = 1 \Rightarrow \cos^2\beta = \frac{1}{5} \Rightarrow \cos\beta = \pm 1/\sqrt{5}$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \left(-\tfrac{3}{5}\right)\left(\tfrac{-\sqrt{5}}{5}\right) - \left(\tfrac{4}{5}\right)\left(\tfrac{-2}{\sqrt{5}}\right)$$

$$= \frac{3\sqrt{5}}{25} + \frac{8\sqrt{5}}{25}$$

$$= \frac{11\sqrt{5}}{25}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \left(\tfrac{4}{5} \cdot -\tfrac{\sqrt{5}}{5}\right) + \left(-\tfrac{3}{5} \cdot \tfrac{-2}{\sqrt{5}}\right)$$

$$= -\frac{4\sqrt{5}}{25} + \frac{6\sqrt{5}}{5\sqrt{5}\sqrt{5}}$$

$$= \frac{\cancel{20} -}{\cancel{25}} \quad \frac{\cancel{\quad} 2\sqrt{5}}{25}$$

ex: Establish the identity

$$\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta} = \cot\alpha\cot\beta + 1$$

$$= \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta}$$

$$= \cot\alpha\cot\beta + 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

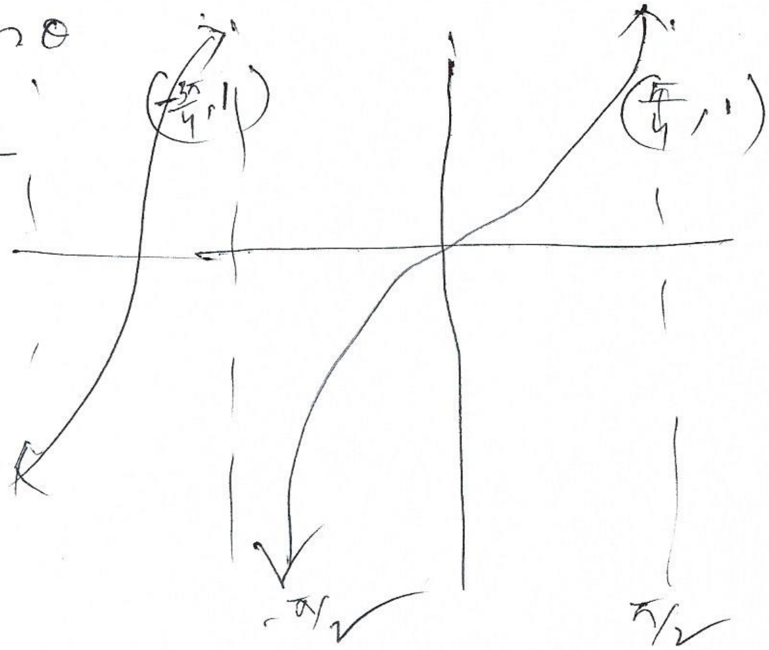
$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\tan(\theta+\pi) = \tan\theta$$

$$= \frac{\tan\theta + \tan\pi}{1 - \tan\theta\tan\pi}$$

$$= \frac{\tan\theta + 0}{1 - \tan\theta \cdot 0}$$

$$= \tan\theta$$



$$\sin\Big(\underbrace{\cos^{-1}\tfrac{1}{2}}_{\alpha} + \underbrace{\sin^{-1}\tfrac{3}{5}}_{\beta}\Big)$$