Toufiqul Hoque
MATH 2312
Pre-Calculus II
Mr. Aguirre

8.4

# Trigonometric Identities

**Def:** Two functions $f, g$ are identically equal if $f(x) = g(x)$

For every value of $x$ which both functions are defined.

Such an equation is called an identity.

$$(x+1)^2 = x^2 + 2x + 1$$

$$\sin x = -\tfrac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}.$$

## Basic trig identities

Quotient: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Reciprocal: $\dfrac{1}{\csc x} = \sin x$

pythagorean: $\cos^2 u + \sin^2 u = 1$

Even-odd: $\cot(-\theta) = -\cot(\theta)$

## Techniques

(I) multiply by a well-chosen

(II) Find a common denominator

(III) Rewrite expression using sine, cosine only.

(IV) factoring.

## Show

$$\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta} \cdot \frac{1-\sin\theta}{\cancel{\cos\theta}\ 1+\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{1+\sin\theta}.$$

$$\frac{1+\sin u}{\sin u} + \frac{\cot u - \cos u}{\cos u}$$

$$= \frac{\cos u + \cancel{\cos u \sin u} + \cot u \sin u - \cancel{\sin u \cos u}}{\sin u \cos u}.$$

$$= \frac{\cos u + \cos u}{\sin u \cos u}$$

$$= \frac{2}{\sin u}.$$

$$= \frac{\cot u}{\csc u}$$

$$= \frac{\frac{\cos u}{\sin u}}{\frac{1}{\sin u}}$$

$$= \cos u$$

ex: simplify

$$\frac{\sin^2 u - 1}{\tan u \sin u - \tan u}$$

$$= \frac{(\sin u + 1)(\sin u - 1)}{\tan u (\sin u - 1)}$$

$$= \frac{\sin u + 1}{\tan u}$$

$$\csc\theta \cdot \tan\theta = \sec\theta$$

$$= \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta .$$

ex: $\sin^2(-\theta) + \cos^2(-\theta) = 1$

$= \left(\sin(-\theta)\right)^2 + \left(\cos(-\theta)\right)^2$

$= \sin^2\theta + \cos^2\theta$

$= 1$

$$\frac{1+\tan\alpha}{1+\cot\alpha}$$

$$= \frac{1+\tan\alpha}{1+\frac{1}{\tan\alpha}}$$

$$= \frac{\tan\alpha+1}{\frac{\tan\alpha+1}{\tan\alpha}}$$

$= \tan\alpha$ .

ex: $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

$$\frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$$

$= 2\operatorname{cosec}\theta$ .